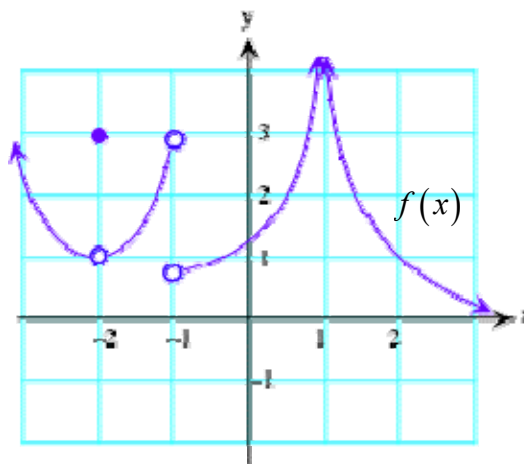


- #1 Evaluate $f(-2)$
 #2 Evaluate $\lim_{x \rightarrow -2} f(x)$
 #3 Evaluate $\lim_{x \rightarrow -1} f(x)$
 #4 Evaluate $\lim_{x \rightarrow 1} f(x)$



- #5 Explain why $f(x)$ is continuous at $x = 2$.

#6 Evaluate $\lim_{x \rightarrow 25} \left[\frac{25-x}{\sqrt{x}-5} \right]$.

#7 Suppose $f(x) = \frac{2x+3}{5x-4}$. Find $f'(2)$ using the limit definition.

#8 Suppose $F(x) = 2x^2 - x$. Find $F'(x)$ using the limit definition.

#9 Suppose $g(x) = \sqrt{x+1}$. Find $g'(x)$ using the limit definition.

#10 Compare the domain of g' to the domain of g from problem #9.

#11 Find $\frac{d}{dx} \left[\frac{2x^2 + 3x + 1}{5x + 3} \right]$. #12 Find $\frac{d}{dx} [e^x x^2]$. #13 Find $\frac{d}{dx} [(3x^7 + 5x^2 - 6x + 1)^{10}]$.

#14 Suppose $y = e^{\sin x}$. Find y' . #15 Suppose $y = \ln(\sin x)$. Find y' .

#16 Suppose $F(x) = \sin^{-1} x$. Evaluate $F'(\sqrt{2}/2)$.

#17 Suppose $f(x) = (10x^2 - x)^{22} (3 + 5x^4)^8$. Find f' using logarithmic differentiation.

#18 Differentiate $xy^3 + \sin(y) = 4x^2 - e^y$ implicitly to find dy/dx .

#19 Recall some trig identities and use the quotient rule as well as the differentiation rules for $\sin x$ and $\cos x$ to prove $(\tan x)' = \sec^2 x$.

#20 Use implicit differentiation to show $(\log_b x)' = \frac{1}{x \cdot \ln b}$.

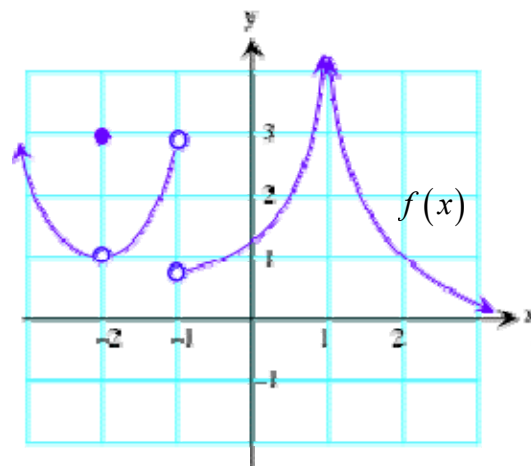
#21 Let $s(t) = 72t - 12t^2$ represent the height in feet of a projectile after t seconds. Determine the velocity and acceleration of the projectile after 2 seconds.

#22 The curve $y = \frac{1}{1+x^2}$ is sometimes called “witch of Agnesi.” Find the tangent to the “witch” when $x = 1$.

#23 State the definition of a linear operator and given an example.

SOLUTIONS on FOLLOWING PAGES

#1 $f(-2)$ refers to the y -value of f when $x = -2$. Thus, $f(-2) = 3$.



#2 $\lim_{x \rightarrow -2} f(x)$ refers to the y -value *approached* by f as x -values approach -2 (from both directions). Therefore, $\lim_{x \rightarrow -2} f(x) = 1$.

#3 $\lim_{x \rightarrow -1^-} f(x)$ refers to the y -value *approached* by f as x -values approach -1 on x -axis from the left. Therefore, $\lim_{x \rightarrow -1^-} f(x) = 3$.

#4 $\lim_{x \rightarrow 1} f(x)$ refers to the y -value *approached* by f as x -values approach zero on x -axis (from the right and left). Therefore, $\lim_{x \rightarrow 1} f(x)$ does not exist (because there is no such value). Since the y -values approach positive infinity, we can write $\lim_{x \rightarrow 1} f(x) = \infty$, which means the limit does not exist but provides extra information.

#5 By definition, f is continuous at a if $\lim_{x \rightarrow a} f(x) = f(a)$. Since $\lim_{x \rightarrow 2} f(x) = 1$ and $f(2) = 1$, f is continuous at 2 by the definition.

#6

$$\lim_{x \rightarrow 25} \left[\frac{25-x}{\sqrt{x}-5} \right] = \lim_{x \rightarrow 25} \left[\frac{(25-x)(\sqrt{x}+5)}{(\sqrt{x}-5)(\sqrt{x}+5)} \right] = \lim_{x \rightarrow 25} \left[\frac{\cancel{(25-x)}^{\cancel{-1}}(\sqrt{x}+5)}{\cancel{x-25}} \right] = \lim_{x \rightarrow 25} (-\sqrt{x}-5) = -\sqrt{25}-5 = -10$$

#7

$$f'(2) = \lim_{h \rightarrow 0} \left(\frac{f(2+h) - f(2)}{h} \right) = \lim_{h \rightarrow 0} \left(\frac{\frac{2(2+h)+3}{5(2+h)-4} - \frac{2(2)+3}{5(2)-4}}{h} \right)$$

$$f'(2) = \lim_{h \rightarrow 0} \left[\left(\frac{7+2h}{6+5h} - \frac{7}{6} \right) \div h \right] = \lim_{h \rightarrow 0} \left[\left(\frac{6(7+2h)}{6(6+5h)} - \frac{7(6+5h)}{6(6+5h)} \right) \cdot \frac{1}{h} \right]$$

$$f'(2) = \lim_{h \rightarrow 0} \left[\left(\frac{42+12h}{6(6+5h)} - \frac{42+35h}{6(6+5h)} \right) \cdot \frac{1}{h} \right] = \lim_{h \rightarrow 0} \left[\left(\frac{42+12h-42-35h}{6(6+5h)} \right) \cdot \frac{1}{h} \right]$$

$$f'(2) = \lim_{h \rightarrow 0} \left[\left(\frac{-23\cancel{h}}{6(6+5h)} \right) \cdot \frac{1}{\cancel{h}} \right] = \lim_{h \rightarrow 0} \left(\frac{-23}{6(6+5h)} \right) = \frac{-23}{6(6+5 \cdot 0)}$$

$$f'(2) = \frac{-23}{36}$$

#8

$$F'(x) = \lim_{h \rightarrow 0} \left[\frac{F(x+h) - F(x)}{h} \right] = \lim_{h \rightarrow 0} \left[\frac{(2(x+h)^2 - (x+h)) - (2x^2 - x)}{h} \right]$$

$$F'(x) = \lim_{h \rightarrow 0} \left[\frac{2(x+h)(x+h) - x - h - 2x^2 + x}{h} \right] = \lim_{h \rightarrow 0} \left[\frac{2(x^2 + 2xh + h^2) - h - 2x^2}{h} \right]$$

$$F'(x) = \lim_{h \rightarrow 0} \left[\frac{2x^2 + 4xh + 2h^2 - h - 2x^2}{h} \right] = \lim_{h \rightarrow 0} \left[\frac{4xh + 2h^2 - h}{h} \right]$$

$$F'(x) = \lim_{h \rightarrow 0} \left[\frac{h(4x + 2h - 1)}{h} \right] = \lim_{h \rightarrow 0} [4x + 2h - 1] = 4x + 2 \cdot 0 - 1$$

$$F'(x) = 4x - 1$$

#9

$$\begin{aligned}g'(x) &= \lim_{h \rightarrow 0} \left[\frac{g(x+h) - g(x)}{h} \right] = \lim_{h \rightarrow 0} \left[\frac{(\sqrt{(x+h)+1}) - (\sqrt{x+1})}{h} \right] \\g'(x) &= \lim_{h \rightarrow 0} \left[\frac{(\sqrt{x+h+1} - \sqrt{x+1}) \cdot (\sqrt{x+h+1} + \sqrt{x+1})}{h \cdot (\sqrt{x+h+1} + \sqrt{x+1})} \right] \\g'(x) &= \lim_{h \rightarrow 0} \left[\frac{(\sqrt{x+h+1})^2 - (\sqrt{x+1})^2}{h(\sqrt{x+h+1} + \sqrt{x+1})} \right] = \lim_{h \rightarrow 0} \left[\frac{x+h+1 - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} \right] \\g'(x) &= \lim_{h \rightarrow 0} \left[\frac{x+h+1 - x - 1}{h(\sqrt{x+h+1} + \sqrt{x+1})} \right] = \lim_{h \rightarrow 0} \left[\frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \right] \\g'(x) &= \lim_{h \rightarrow 0} \left[\frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \right] = \frac{1}{\sqrt{x+0+1} + \sqrt{x+1}} = \frac{1}{\sqrt{x+1} + \sqrt{x+1}} \\g'(x) &= \frac{1}{2\sqrt{x+1}}\end{aligned}$$

#10 The domain of g is $[-1, \infty)$, but the domain of g' is $(-1, \infty)$. The one x -value that is not in the domain of g' but is in the domain of g represents a point where g is not differentiable. In other words, g is non-differentiable at $x = -1$.

#11

$$\frac{d}{dx} \left[\frac{2x^2 + 3x + 1}{5x + 3} \right] = \frac{(5x + 3) \cdot (4x + 3) - (2x^2 + 3x + 1) \cdot 5}{(5x + 3)^2} = \frac{20x^2 + 27x + 9 - 10x^2 - 15x - 5}{(5x + 3)^2} = \frac{10x^2 + 12x + 4}{(5x + 3)^2}$$

$$\#12 \quad \frac{d}{dx} [e^x x^2] = e^x \cdot 2x + x^2 \cdot e^x = x e^x (2 + x)$$

$$\#13 \quad \frac{d}{dx} [(3x^7 + 5x^2 - 6x + 1)^{10}] = 10(3x^7 + 5x^2 - 6x + 1)^9 \cdot (21x^6 + 10x - 6)$$

$$\#14 \quad y = e^{\sin x} \Rightarrow y' = e^{\sin x} \cdot \cos x$$

$$\#15 \quad y = \ln(\sin x) \Rightarrow y' = \cot x$$

$$\#16 \quad F(x) = \sin^{-1} x \Rightarrow F'(x) = \frac{1}{\sqrt{1-x^2}}$$

∴

$$F'(\sqrt{2}/2) = \frac{1}{\sqrt{1-(\sqrt{2}/2)^2}}$$

$$F'(\sqrt{2}/2) = \frac{1}{\sqrt{1-\frac{2}{4}}}$$

$$F'(\sqrt{2}/2) = \frac{1}{\sqrt{\frac{1}{2}}}$$

$$F'(\sqrt{2}/2) = \frac{1}{\frac{1}{\sqrt{2}}}$$

$$F'(\sqrt{2}/2) = \sqrt{2}$$

#17

$$f(x) = (10x^2 - x)^{22} (3 + 5x^4)^8$$

$$\ln f(x) = \ln(10x^2 - x)^{22} (3 + 5x^4)^8$$

$$\ln f(x) = 22 \ln(10x^2 - x) + 8 \ln(3 + 5x^4)$$

$$\frac{f'(x)}{f(x)} = 22 \cdot \frac{20x-1}{10x^2-x} + 8 \cdot \frac{20x^3}{3+5x^4}$$

$$\cancel{f(x)} \cdot \frac{f'(x)}{\cancel{f(x)}} = f(x) \cdot \left[22 \cdot \frac{20x-1}{10x^2-x} + 8 \cdot \frac{20x^3}{3+5x^4} \right]$$

$$f'(x) = (10x^2 - x)^{22} (3 + 5x^4)^8 \left[22 \cdot \frac{20x-1}{10x^2-x} + 8 \cdot \frac{20x^3}{3+5x^4} \right]$$

#18

$$xy^3 + \sin(y) = 4x^2 - e^y$$

$$\frac{d}{dx}[xy^3 + \sin(y)] = \frac{d}{dx}[4x^2 - e^y]$$

$$\frac{d}{dx}[xy^3] + \frac{d}{dx}[\sin(y)] = \frac{d}{dx}[4x^2] + \frac{d}{dx}[e^y]$$

$$x \cdot \frac{d}{dx}[y^3] + y^3 \cdot \frac{d}{dx}[x] + \cos(y) \cdot \frac{dy}{dx} = 8x + e^y \cdot \frac{dy}{dx}$$

$$x \cdot 3y^2 \cdot \frac{dy}{dx} + y^3 + \cos(y) \cdot \frac{dy}{dx} = 8x + e^y \cdot \frac{dy}{dx}$$

$$3xy^2 \cdot \frac{dy}{dx} + \cos(y) \cdot \frac{dy}{dx} - e^y \cdot \frac{dy}{dx} = 8x - y^3$$

$$\frac{dy}{dx}(3xy^2 + \cos(y) - e^y) = 8x - y^3$$

$$\frac{dy}{dx} = \frac{8x - y^3}{3xy^2 + \cos(y) - e^y}$$

#19

$$\underbrace{\frac{d}{dx}[\tan x]}_{\text{fundamental identity}} = \frac{d}{dx} \left[\frac{\sin x}{\cos x} \right] = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\underbrace{\cos^2 x}_{\text{quotient rule}}} = \frac{\overbrace{\cos^2 x + \sin^2 x}^{\text{Pythagorean Identity}}}{\cos^2 x} = \frac{1}{\cos^2 x} = \frac{1}{\underbrace{\cos x \cos x}_{\text{reciprocal identity}}} = \sec^2 x \quad \blacksquare$$

#20 Recall the definition of a logarithm: $y = \log_b x \Rightarrow b^y = x$. Using implicit differentiation on $b^y = x$, we obtain the following.

$$b^y = x$$

$$\frac{d}{dx}(b^y) = \frac{d}{dx}(x)$$

On the left-hand side, recall the differentiation rule $\frac{d}{dx}(b^x) = b^x \cdot \ln b$. On the right, use the power rule.

$$\frac{d}{dx}(b^y) = \frac{d}{dx}(x)$$

$$b^y \cdot \ln b \cdot y' = 1$$

$$y' = \frac{1}{b^y \cdot \ln b}$$

Recall that $b^y = x$ and substitute to obtain y' in terms of x .

$$y' = \frac{1}{x \cdot \ln b} \quad \blacksquare$$

#21 $s(t) = 72t - 12t^2$
 $v(t) = 72 - 24t$
 $a(t) = -24$
 \therefore
 $v(2) = 24 \frac{\text{ft}}{\text{sec}}$
 $a(2) = -24 \frac{\text{ft}}{\text{sec}^2}$

#22 $y = \frac{1}{1+x^2} \Rightarrow y' = \frac{-2x}{(1+x^2)^2}$
 point of tangency: $y(1) = \frac{1}{1+(1)^2} = \frac{1}{2}$
 slope of tangent: $y'(1) = \frac{-2(1)}{(1+(1)^2)^2} = -\frac{1}{2}$
 equation of tangent: $y = -\frac{1}{2}x + 1$

#23 Let c be a scalar. Let T be an operator with valid arguments θ and λ . Then, T is a linear operator if the following two statements are true: $T(\theta + \lambda) = T(\theta) + T(\lambda)$ and $T(c \cdot \theta) = c \cdot T(\theta)$. Both limits and differentiation are linear operators.