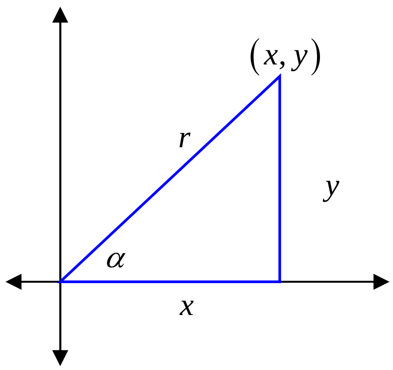
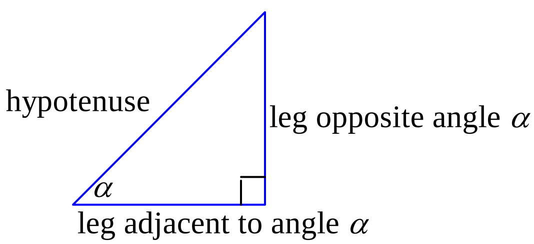
Right Triangle Trigonometry

In the previous lecture, we said that the inputs of the trigonometric functions are *usually* thought of as interior angles of a right triangle, but we decided to define the functions in a less restrictive way. For instance, we said if (x, y) is any point other than the origin on the terminal side of an angle α in standard position and $r = \sqrt{x^2 + y^2}$ (i.e., r is the radius of a circle centered at the origin), then $\sin\alpha = y/r$. This definition allows α to take any real value, which was our goal, but in its own way this definition can be restrictive since it ties the trigonometric functions to a coordinate system.



In this lecture, we will take away the coordinate system and regard the values of the trigonometric functions simply as ratios of the lengths of the sides of a right triangle.



Hence, using the right triangle above, we have the following theorem.

If α is an acute interior angle of a right triangle, then

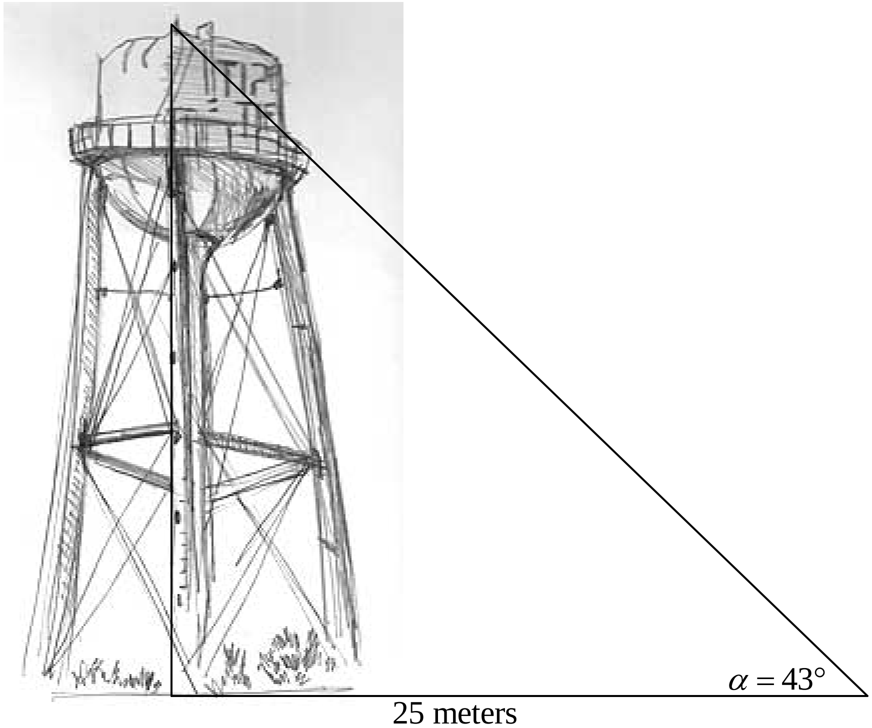
$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

Since cosecant, secant, and cotangent are the reciprocals of sine, cosine, and tangent respectively, we can use the acute angle of a right triangle to calculate the values of all six trigonometric functions.

Using right triangle trigonometry, we can find the height of an object without actually measuring the object. Consider the water tower below. Suppose a surveyor stands 25 meters to the side of the tower and measures the angle of elevation α to the top of the tower at $43°$.



In this situation, calculating the height of the tower is a simple task as follows.

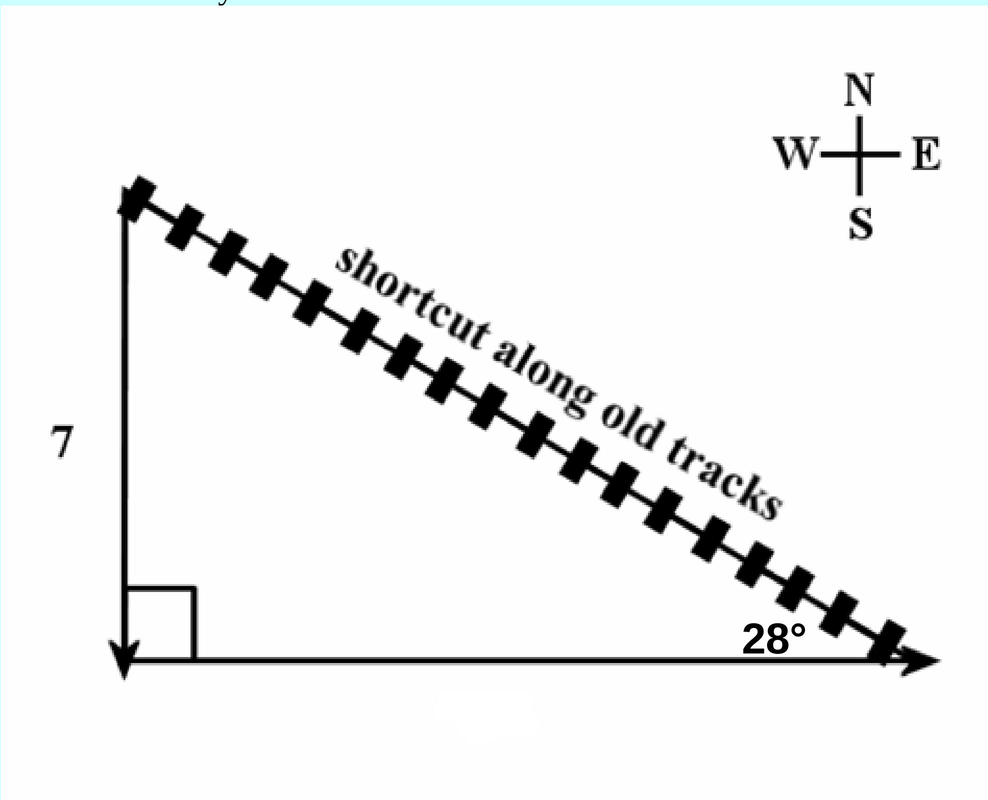
$$\tan(43°) = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan(43°) = \frac{\text{height}}{25 \text{ meters}}$$
$$25 \text{ meters} \times \tan(43°) = \text{height}$$

Using a table of trigonometric values (i.e., our calculator in degree mode), we see that $\tan(43°) \approx 0.932515$, so we conclude below.

$$25 \text{ meters} \times \tan(43°) = \text{height}$$
$$25 \text{ meters} \times 0.932515 \approx \text{height}$$
$$23.3 \text{ meters} \approx \text{height}$$

Example Exercise

To get home from school, Tameka can follow the roads seven miles south then even farther due east, or she can take a shortcut following the old railroad tracks. How many miles does Tameka walk if she takes the shortcut?



Using a table of values for sine, i.e., our calculator in degree mode, we see that $\sin(28°) \approx 0.46947$. Using the right triangle theorem for the trigonometric functions, we let s represent the length of the shortcut and solve as below.

$$\sin(\theta) = \frac{\text{opposite side length}}{\text{hypotenuse length}}$$

$$\sin(28°) = \frac{\text{7 miles}}{\text{shortcut length}}$$

$$0.46947 \approx \frac{\text{7 miles}}{s}$$

$$s \times 0.46947 \approx \text{7 miles}$$

$$\frac{s \times \cancel{0.46947}}{\cancel{0.46947}} \approx \frac{\text{7 miles}}{0.46947}$$

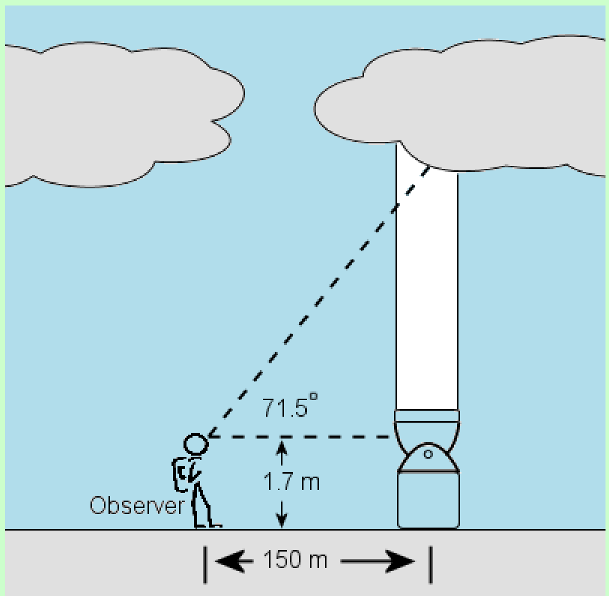
$$s \approx 14.9 \text{ miles}$$

Suggested Homework

Section 5.6: #9-13 odd, #23, #24, #41, #43, #51

Application Exercise

In meteorology, the vertical distance from the ground to the base of the clouds is called the *ceiling*. To measure the ceiling, weather watchers directed a spotlight vertically overhead. An observer made the measurements shown in the figure below.



What is the ceiling?