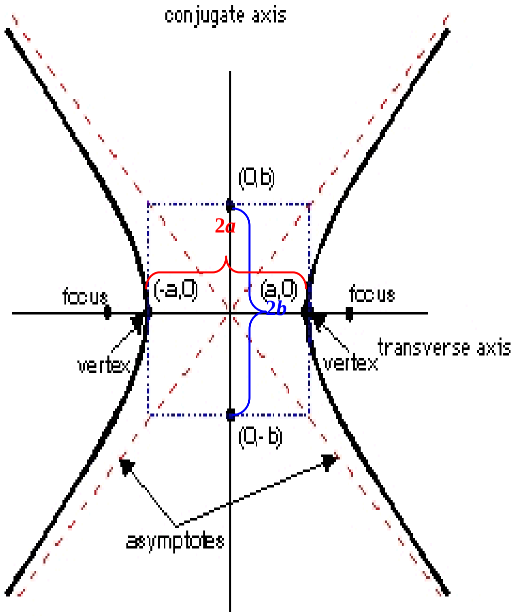
Hyperbolas

A hyperbola has a center (h,k) and two important axes, the conjugate and transverse axes. The transverse axis connects the vertices of the hyperbola. The conjugate axis is perpendicular to the transverse axis. If the distance from the center to one of the vertices is *a* and the distance from the center to one of the foci is *c*, the length of the conjugate axis is said to have a length of 2*b* where $c^2 = a^2 + b^2$. The transverse and conjugate axes form a rectangle with dimensions $a \times b$. This rectangle, the fundamental rectangle, has corner points that share common *x*-coordinates with the vertices if the transverse axis is horizontal and common *y*-coordinates with the vertices if the transverse axis is vertical.



The following box contains the equation of a hyperbola.

A hyperbola centered at (h,k) with horizontal transverse axis of length 2*a* and vertical conjugate axis of length 2*b* has the equation below.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

This hyperbola has foci at $(h \pm c, k)$ where $c^2 = a^2 + b^2$.

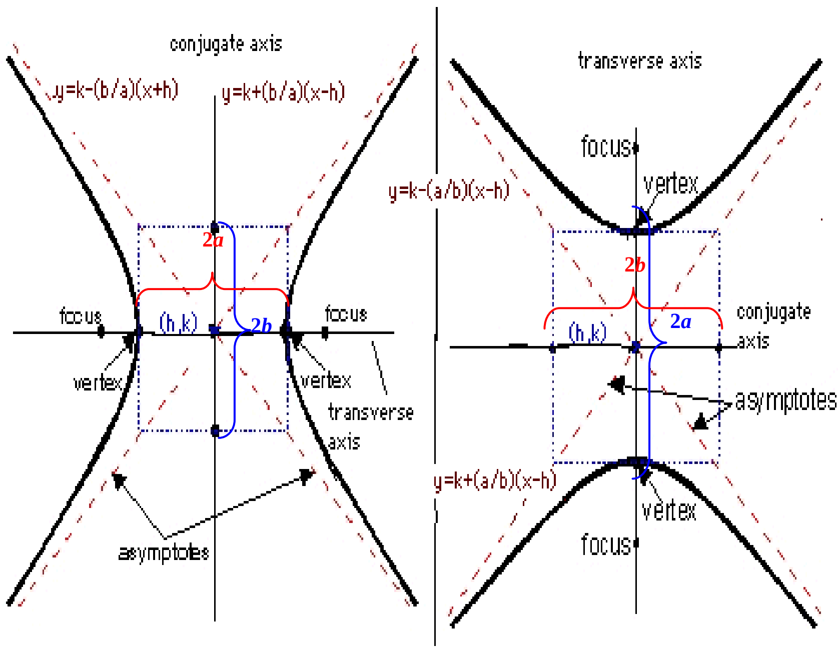
Similarly, a hyperbola centered at (h,k) with vertical transverse axis of length 2*a* and horizontal conjugate axis of length 2*b* has the equation below.

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

This hyperbola has foci at $(h, k \pm c)$ where $c^2 = a^2 + b^2$.

Hyperbola approach a pair of asymptotes. For hyperbola with a horizontal transverse axis the points on the hyperbola approach the lines $y = k \pm \frac{b}{a}(x-h)$ as $|x|$ approaches infinity. These lines are asymptotes of the hyperbola and extended diagonals of the fundamental rectangle. If the transverse axis is vertical the asymptotes will have the same equations with reciprocal slopes $y = k \pm \frac{a}{b}(x-h)$.

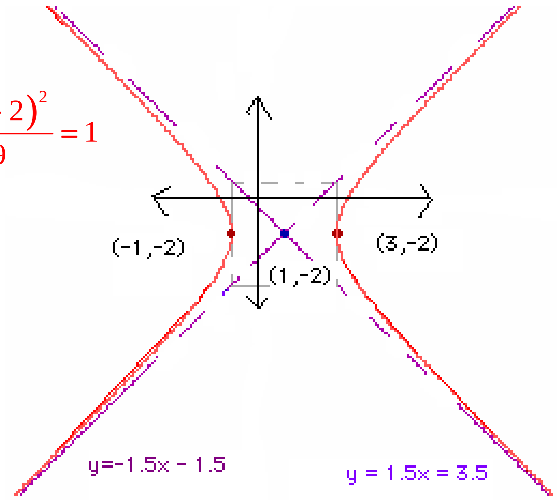
The diagram below summarizes the important aspects of a hyperbola.



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

To graph a hyperbola it is convenient to know the dimensions, $a \times b$, of the fundamental rectangle and to know its center because the asymptotes pass through the vertices of the fundamental rectangle and the center. It is also important to find the foci since the definition of the hyperbola depends upon the focal points. Consider the hyperbola given by the equation $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{9} = 1$. The transverse axis is horizontal. The center is (1,–2). The vertices are a-units away from the center along the transverse axis, so they occur at (3,–2) and (–1,–2). The foci are c-units away from the center along the transverse axis. Recall that $c^2 = a^2 + b^2$, so $c^2 = 13$ and $c = \sqrt{13}$. The foci, therefore, occur at $(1+\sqrt{13}, -2)$ and $(1-\sqrt{13}, -2)$. The asymptotes are the lines $y = -2 + \frac{3}{2}(x-1)$ and $y = -2 - \frac{3}{2}(x-1)$. The graph appears on the following page.

$$\frac{(x-1)^2}{4}-\frac{(y+2)^2}{9}=1$$

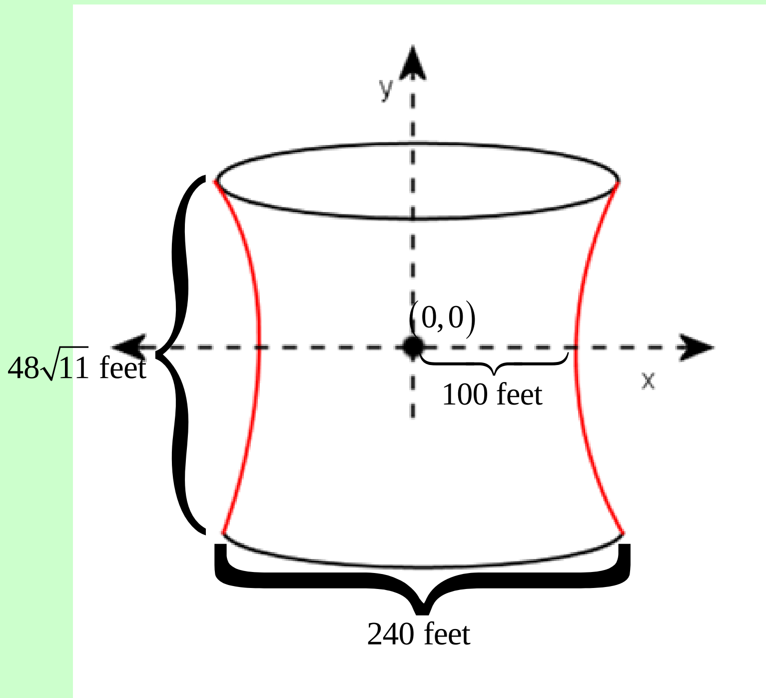


Suggested Homework

Section 10.3: #19-27 odd, #35, #47

Application Exercise

A cooling tower for a nuclear power plant has a hyperbolic cross section as shown in the diagram below.



The diameter of the tower at the top and bottom is 240 feet while the diameter in the middle is 200 feet. The height of the tower equals $48 \cdot \sqrt{11}$ feet. Find the equation of the hyperbola, using the coordinate system shown in the diagram.