College Algebra

Instruction: *The Irrational Number* e

The quantity denoted by the symbol *e* has special significance in mathematics. It is an irrational number that is the base of natural logarithms (to be discussed in the next lecture), and it is often seen in real world problems that involve natural exponential growth or decay. The approximated value of *e* is 2.7182818284 . . .

Students can replicate this value by using the *e*^ key on a graphing calculator and entering 1 as the exponent. Often a *2nd* key must be used to generate *e*^. The housetop symbol, ^ , represents the fact that *e* will be raised to some power. Since *e* is often involved in exponential growth or decay problems, it is usually raised to some power when used for calculations; thus, the calculator anticipates the need to apply an exponent to *e*. Raising *e* to the first power replicates the approximated value of *e*.

Students can also generate approximations of *e* by substituting extremely large numbers for *m* into the following expression:

$$\left(1+\frac{1}{m}\right)^m$$

This expression approaches *e* as the variable *m* approaches infinity.

Functions of the form $y = ae^{kx}$ or $y = ae^{-kx}$ where *a* and *k* are real number non-zero constants are often used in science problems.

The following properties of exponents can be used to simplify expressions with *e*.

$$a^1 = a;\ a^0 = 1;\ \left(a^r\right)^s = a^{r\cdot s};\ a^r a^s = a^{r+s};\ \frac{a^r}{a^s} = a^{r-s};\ \text{and}\ a^{-r} = \frac{1}{a^r}$$

For example, consider $\dfrac{e^{9x}+e^{6x}+e^{3x}}{\left(e^x\right)^2 e^x}$.

$$\frac{e^{9x}+e^{6x}+e^{3x}}{\left(e^x\right)^2 e^x} = \frac{e^{3x}\left(e^{9x-3x}+e^{6x-3x}+e^{3x-3x}\right)}{e^{x\cdot 2}e^x}$$

$$= \frac{e^{3x}\left(e^{6x}+e^{3x}+e^{0}\right)}{e^{2x+x}}$$

$$= \frac{\cancel{e^{3x}}\left(e^{6x}+e^{3x}+1\right)}{\cancel{e^{3x}}}$$

$$= e^{6x}+e^{3x}+1$$

Example Exercises 3.5

Instruction: *The Irrational Number* e

Example 1
The Irrational Number *e*

Evaluate $e^{17} \cdot e^{-17}$ without using a calculator. Round the answer to the nearest thousandths.

Recall the exponent property of real numbers, $a^r \cdot a^s = a^{r+s}$: $e^{17} \cdot e^{-17} = e^{17+(-17)} = e^0 = 1$.

Example 2
The Irrational Number *e*

Evaluate $\frac{e^8}{e^7}$ without using a calculator. Round the answer to the nearest thousandths.

Recall the exponent property of non-zero real numbers, $\frac{a^r}{a^s} = a^{r-s}$: $\frac{e^8}{e^7} = e^{8-7} = e^1 = e \approx 2.718$.

Example 3
The Irrational Number *e*

Evaluate $\frac{\left(25e^2\right)^{\frac{1}{2}}}{5e}$ without using a calculator.

Recall the exponent property of real numbers, $\left(a^r b^s\right)^t = a^{r \cdot t} b^{s \cdot t}$.

$$\frac{\left(25e^2\right)^{\frac{1}{2}}}{5e} = \frac{25^{\frac{1}{2}} e^{\cancel{2} \cdot \frac{1}{\cancel{2}}}}{5e} = \frac{25^{\frac{1}{2}} e}{5e}$$

Reduce appropriately and recall the exponent property $a^{\frac{1}{r}} = \sqrt[r]{a}$.

$$\frac{25^{\frac{1}{2}} \cancel{e}}{5 \cancel{e}} = \frac{\sqrt{25}}{5} = \frac{5}{5} = 1$$

Example 4
The Irrational Number *e*

Evaluate $\dfrac{\left(e^5\right)^3}{e^7 \cdot e^6}$. Round the answer to the nearest thousandth.

Recall the exponent properties of real numbers, $a^r \cdot a^s = a^{r+s}$ and $\left(a^r\right)^s = a^{r \cdot s}$.

$$\frac{\left(e^5\right)^3}{e^7 \cdot e^6} = \frac{e^{5 \cdot 3}}{e^{7+6}} = \frac{e^{15}}{e^{13}}$$

Recall the exponent property of non-zero real numbers, $\dfrac{a^r}{a^s} = a^{r-s}$.

$$\frac{e^{15}}{e^{13}} = e^{15-13} = e^2$$

Use the calculator e^x key to evaluate e^2. Round to the nearest thousandth.

$$e^2 \approx 7.389$$

Example 5
The Irrational Number *e*

Simplify $\dfrac{\frac{e^5}{e^6}}{\frac{e^5}{e^3}}$. Leave all answers exact.

Recall that division with fractions is multiplication of the dividend by the reciprocal of the divisor.

$$\frac{\frac{e^5}{e^6}}{\frac{e^5}{e^3}} = \frac{e^5}{e^6} \div \frac{e^5}{e^3} = \frac{\cancel{e^5}}{e^6} \cdot \frac{e^3}{\cancel{e^5}} = \frac{e^3}{e^6}.$$

Recall the exponent property of non-zero real numbers, $\dfrac{a^r}{a^s} = \dfrac{1}{a^{s-r}}$.

$$\frac{e^3}{e^6} = \frac{1}{e^{6-3}} = \frac{1}{e^3}$$

Example 6
The Irrational Number *e*

Simplify $\dfrac{e^{5x} - e^{4x} + 2e^{2x}}{e^x \cdot e^x}$.

Recall the exponent property of real numbers, $a^r \cdot a^s = a^{r+s}$ and simplify the denominator.

$$e^x \cdot e^x = e^{x+x} = e^{2x}.$$

Factor the greatest common factor in the numerator.

$$\frac{e^{5x} - e^{4x} + 2e^{2x}}{e^{2x}} = \frac{e^{2x}\left(e^{5x-2x} - e^{4x-2x} + 2e^{2x-2x}\right)}{e^{2x}} = \frac{e^{2x}\left(e^{3x} - e^{2x} + 2e^{0}\right)}{e^{2x}}$$

Simplify and reduce.

$$\frac{\cancel{e^{2x}}\left(e^{3x} - e^{2x} + 2e^{0}\right)}{\cancel{e^{2x}}} = e^{3x} - e^{2x} + 2 \cdot 1 = e^{3x} - e^{2x} + 2$$

Practice Set 3.5

Evaluate the following expressions. Round to the nearest thousandth when necessary. Recall the properties of exponents: $a^1 = a$; $a^0 = 1$; $(a^r)^s = a^{r \cdot s}$; $a^r a^s = a^{r+s}$; and $\frac{a^r}{a^s} = a^{r-s}$.

#1 $\frac{e^7}{e^6}$ #2 $\left(e^{\frac{1}{3}}\right)^6$ #3 $\frac{e^x - e^{x+1}}{e^x}$ #4 $e^5 e^{-5}$ #5 $e^{7.5} e^{-6.5}$

Simplify the following expressions. Keep all answers exact. Recall the properties of exponents: $a^1 = a$; $a^0 = 1$; $(a^r)^s = a^{r \cdot s}$; $a^r a^s = a^{r+s}$; $a^{-r} = \frac{1}{a^r}$; and $\frac{a^r}{a^s} = a^{r-s}$.

#6 $e^2 e^x$ #7 $\frac{e^x - e^{1+x} - 2e^{x+1}}{e^x}$ #8 $\frac{e^{2x} - e^{3x} - 2e^{2x}}{(e^x)^2}$

ANSWERS
#1) 2.718
#2) 7.389
#3) –1.718
#4) 1
#5) 2.718
#6) e^{2+x}
#7) $1 - 3e$
#8) $-1 - e^x$

Study Exercise 3.5

Problems

Recall the exponent properties: $a^1 = a$; $a^0 = 1$; $(a^r)^s = a^{r \cdot s}$; $a^r a^s = a^{r+s}$; $\frac{a^r}{a^s} = a^{r-s}$; & $a^{-r} = \frac{1}{a^r}$.

Evaluate the following expressions. Round to the nearest thousandth when necessary.

#1 $\left(e^{\frac{4}{5}}\right)^{\frac{5}{4}}$

#2 $\frac{e^{\frac{7}{3}}}{e^{\frac{4}{3}}}$

Simplify the following expressions. Leave all answers exact.

#3 $\frac{e^{6x} - e^{4x} + e^{2x}}{(e^x)^2}$

#4 $e^2 \cdot e^{-2}$

College Algebra

Instruction: *Logarithms*

A *logarithm* is the power to which some fixed number, called a *base*, must be raised in order to obtain a specified number called the *argument*. Logarithms of specified numbers will have different values, depending on which base is used. The three most commonly used bases are 2, 10, and e. Scientific calculators can evaluate the logarithm of numbers using base 10 and base e.

To introduce the student to logarithms, this lecture will begin by considering logarithms in exponential form. Consider the exponential function below:

$$x = b^y$$

In this equation, b raised to the y power yields x. The number b, therefore, is the base and y is the logarithm (base b) of x. This equation can be written alternatively:

$$y = \log_b x$$

This form of the equation depicts y as the logarithm (base b) of x (the specified number or *argument*).

Evaluating logarithmic expressions sometimes requires a calculator. For introductory purposes, this lecture will evaluate logarithms that do not require a calculator. For example:

$$\log_2 8$$

This expression reads, "logarithm (base two) of eight." The logarithm is 3 because $2^3 = 8$.

Mathematicians use special notation for common logarithms--logarithms (base 10)--and for natural logarithms--logarithms (base e). Usually the logarithm's base is written as a subscript to the "log" symbol. With common logarithms the base is not written but simply understood to be ten. Thus $\log 100$ reads, "logarithm (base ten) of one hundred." With natural logarithms, the "log" symbol is replaced by "ln" and the base is understood to be e. Thus, $\ln 2$ reads, "logarithm (base e) of two" or "natural log of two."

Further examples:

1) $\log_3 81$

Logarithm to the base 3 of 81 is 4 because $3^4 = 81$. Therefore, $\log_3 81 = 4$.

2) $\log_\pi 1$

Logarithm to the base π of 1 is 0 because $\pi^0 = 1$. Therefore, $\log_\pi 1 = 0$.

3) $\log_4 2$

Logarithm to the base 4 of 2 is $\frac{1}{2}$ because $\sqrt{4} = 2$. Therefore, $\log_4 2 = \frac{1}{2}$.

4) $\ln e$

Natural log of e is 1 because $e^1 = e$. Therefore, $\ln e = 1$.

5) $\log .01$

Logarithm to the base 10 of .01 is –2 since $10^{-2} = \frac{1}{100}$. Therefore, $\log .01 = -2$.

Example Exercises 3.6

Instruction: *Introduction to Logarithms*

Example 1
Evaluating Logarithms without a Calculator

Evaluate $\log_4\left(\frac{1}{16}\right)$ without using a calculator.

Recognize the argument $1/16$ as a power of the base 4.

$$\frac{1}{16} = \frac{1}{4^2} = 4^{-2}$$

Since $\frac{1}{16} = 4^{-2}$, $\log_4\left(\frac{1}{16}\right) = -2$.

Example 2
Evaluating Logarithms without a Calculator

Evaluate $\log_{\frac{1}{5}}(0.0016)$ without using a calculator.

Convert the argument 0.0016 to a fraction and reduce.

$$0.0016 = \frac{16}{10,000} = \frac{\cancel{2^4}}{\cancel{2^4} \cdot 5^4} = \frac{1}{5^4}$$

Recognize the argument $1/5^4$ as a power of the base $1/5$.

$$0.0016 = \frac{1}{5^4} = \left(\frac{1}{5}\right)^4$$

Since $0.0016 = \left(\frac{1}{5}\right)^4$, $\log_{\frac{1}{5}}(0.0016) = 4$.

Example 3
Evaluating Logarithms without a Calculator

Evaluate $\log_{64}(4)$ without using a calculator.

Recognize the argument 4 as a root (fractional power) of the base 64.

$$4 = \sqrt[3]{64} = 64^{\frac{1}{3}}$$

Since $4 = 64^{\frac{1}{3}}$, $\log_{64}(4) = \dfrac{1}{3}$.

Example 4
Evaluating Logarithms without a Calculator

Evaluate $\log_{16}(64)$ without using a calculator.

Recognize the argument 64 and the base 16 as powers of 4.

$$4^2 = 16$$
$$4^3 = 64$$

Rewrite the argument as a power of 16.

$$64 = 4^3 = \left(\sqrt{16}\right)^3 = \left(4^{\frac{1}{2}}\right)^3 = 4^{\frac{3}{2}}$$

Since $64 = 4^{\frac{3}{2}}$ $4 = 64^{\frac{1}{3}}$, $\log_{16}(64) = \dfrac{3}{2}$.

Practice Set 3.6

Evaluate the following expressions.

#1 $\log_2 8$

#2 $\log_3 9$

#3 $\log \frac{1}{10}$

#4 $\log_3 81$

#5 $\log_9 27$

#6 $\ln e^{4.5}$

#7 $\log 10^6$

#8 $\log_4 2$

#9 $\log_2 2^x$

#10 $\log_{125} 5$

#11 $\log_7 7^2$

#12 $\log_3 9^2$

<u>ANSWERS</u>

#1) 3
#2) 2
#3) –1
#4) 4
#5) $^3/_2$
#6) 4.5
#7) 6
#8) ½
#9) x
#10) ⅓
#11) 2
#12) 4

Study Exercise 3.6

Problems

Evaluate the following logarithms.

#1 $\log_3 81$

#2 $\log_5 \sqrt{5}$

#3 $\log_2 \frac{1}{32}$

#4 $\log 100$

College Algebra

Instruction: Logarithm Properties

	Logarithm Properties (The numbers *x* & *y* are two real numbers both greater than zero, *b* is a base, *p* is any real number.)	Example 1	Example 2
A	$\log_b b^x = x$	$\log_2 2^3 = 3$	$\ln e^9 = 9$
B	$\log_b xy = \log_b x + \log_b y$	$\log[x(x+1)] = \log x + \log(x+1)$	$\log_2 16 = \log_2 4 + \log_2 4$
C	$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\ln\left(\frac{x+1}{x}\right) = \ln(x+1) - \ln x$	$\log_2 16 = \log_2 32 - \log_2 2$
D	$\log_b x^p = p \cdot \log_b x$	$\log_b \sqrt{x} = \frac{1}{2} \cdot \log_b x$	$\log_2 5^3 = 3 \cdot \log_2 5$
E	$\log_{b_1} x = \frac{\log_{b_2} x}{\log_{b_2} b_1}$	$\log_b a = \frac{\ln a}{\ln b}$	$\log_8 4 = \frac{\ln 4}{\ln 8}$
F	$b^{\log_b x} = x$	$2^{\log_2 6} = 6$	$e^{\ln 7} = 7$

The properties above can be used to expand a single logarithm to a sum or difference of logarithms (using properties B and C) or a ratio of logarithms (using property E) or a product of two factors, one a number, the other a logarithm (property D). The properties can also simplify a logarithm to a number (property A) or simplify an exponential expression to a number (property F).

Instruction: *Approximating the Natural Logarithm Function*

Consider the natural logarithm function $f(x) = \ln(x)$. Like all logarithmic functions, $f(x)$ takes rational values only when its argument equals a rational power of the base. The base *e*, however, is, itself, an irrational number approximately equal to 2.718, so one (which is equal to e^0) is the only integer argument for which $f(x)$ will take a rational number value. Comparing a table of values of $f(x)$ to a logarithmic function with a rational base illustrates this observation. Consider the table of values below comparing $f(x) = \ln(x)$ to $g(x) = \log_2(x)$.

x	ln(*x*)	$\log_2(x)$
⅛	–2.079 . . .	–3
¼	–1.386 . . .	–2
½	–0.693 . . .	–1
1	0	0
2	0.6931 . . .	2
3	1.0986 . . .	1.585 . . .
4	1.3863 . . .	2

Note that the values of ln(x) are all irrational numbers (other than when $x = 1$) while the values of $\log_2(x)$ are rational numbers when the argument (in this case, the input value) is a power of two. Since ln(x) takes irrational values for integer inputs, a quick means for approximating ln(x) would be useful, and the multiplicative property of logarithms provides one such means.

The multiplicative property of logarithms states that the log of the pth power of a number is p times the log of that number, in symbols, $\log_b n^p = p \cdot \log_b n$. Consider that $\ln(2) \approx 0.6931$. Note that the natural log of four equals $\ln 2^2$, which, in turn, equals $2\ln 2$, which is approximately 2(0.6931) or 1.3862. Accordingly, various values of ln(x) can approximated as multiples of 0.6931. For example, $\ln(8) = \ln 2^3 = 3\ln 2 \approx 3(0.6931) \approx 2.0793$. Note further that since $\ln(4) \approx 1.3862$ and $\ln(8) \approx 2.0793$ then ln(5), ln(6), and ln(7) all take values between 1.3862 and 2.0793.

This method of approximating ln(x) becomes more efficient after considering arguments that are powers of three. Since the natural log of three approximately equals 1.1, values of ln(x) can be approximated as multiples of 1.1 when x equals a power of three, so $\ln(9) \approx 2.2$ and $\ln(27) \approx 3.3$. Moreover, since $\ln(9) \approx 2.2$ and $\ln(27) \approx 3.3$, then ln(x) takes on values between 2.2 and 3.3 for all x-values between 9 and 27.

Instruction: ***Properties of Logarithms***

Example 1
Expanding a Logarithm into the Product of a Number and a Logarithm

Rewrite $\log_b\left(a^3\right)$ as a product of a number and a logarithm.

Apply the property of logs that states $\log_b\left(M^p\right) = p \cdot \log_b\left(M\right)$.

$$\log_b\left(a^3\right) = 3\log_b\left(a\right)$$

Example 2
Expanding a Logarithm into a Sum or Difference of Logarithms

Expand $\log_b\left(\frac{x^2}{y^3}\right)$ as a sum or difference of logarithms. Remove all exponents from the argument.

Apply the property of logs that states $\log_b\left(M/N\right) = \log_b M - \log_b N$.

$$\log_b\left(\frac{x^2}{y^3}\right) = \log_b\left(x^2\right) - \log_b\left(y^3\right)$$

Apply the property of logs that states $\log_b\left(M^p\right) = p \cdot \log_b\left(M\right)$.

$$\log_b\left(x^2\right) - \log_b\left(y^3\right) = 2\log_b\left(x\right) - 3\log_b\left(y\right)$$

Example 3
Evaluating Logarithms without a Calculator

Expand $\log_b\left[\left(\frac{a^2b}{cd}\right)^3\right]$ as a sum or difference of logarithms. Remove all exponents from the argument.

Apply the property of logs that states $\log_b\left(M^p\right) = p\cdot\log_b\left(M\right)$.

$$\log_b\left[\left(\frac{a^2b}{cd}\right)^3\right] = 3\cdot\log_b\left(\frac{a^2b}{cd}\right)$$

Apply the property of logs that states $\log_b\left(M/N\right) = \log_b\left(M\right) - \log_b\left(N\right)$.

$$3\cdot\log_b\left(\frac{a^2b}{cd}\right) = 3\left[\log_b\left(a^2b\right) - \log_b\left(cd\right)\right]$$

Apply the property of logs that states $\log_b\left(MN\right) = \log_b\left(M\right) + \log_b\left(N\right)$.

$$\begin{aligned}3\left[\log_b\left(a^2b\right) - \log_b\left(cd\right)\right] &= 3\left[\log_b\left(a^2\right) + \log_b\left(b\right) - \left(\log_b\left(c\right) + \log_b\left(d\right)\right)\right]\\ &= 3\left[\log_b\left(a^2\right) + \log_b\left(b\right) - \log_b\left(c\right) - \log_b\left(d\right)\right]\end{aligned}$$

Apply the property of logs that states $\log_b\left(M^p\right) = p\cdot\log_b\left(M\right)$.

$$3\left[\log_b\left(a^2\right) + \log_b\left(b\right) - \log_b\left(c\right) - \log_b\left(d\right)\right] = 3\left[2\log_b\left(a\right) + \log_b\left(b\right) - \log_b\left(c\right) - \log_b\left(d\right)\right]$$

Apply the property of logs that states $\log_b b^p = p$.

$$3\left[2\log_b\left(a\right) + \log_b\left(b\right) - \log_b\left(c\right) - \log_b\left(d\right)\right] = 3\left[2\log_b\left(a\right) + 1 - \log_b\left(c\right) - \log_b\left(d\right)\right]$$

Distribute.

$$3\left[2\log_b\left(a\right) + 1 - \log_b\left(c\right) - \log_b\left(d\right)\right] = 6\log_b\left(a\right) + 3 - 3\log_b\left(c\right) - 3\log_b\left(d\right)$$

Example 4
Rewriting Sums and Differences of Logarithms as Single Logarithms

Rewrite $\log_b(x+5)+\log_b(x-5)$ as a single logarithm.

Apply the property of logs that states $\log_b(MN)=\log_b(M)+\log_b(N)$.

$$\begin{aligned}\log_b(x+5)+\log(x-5) &= \log_b\left[(x+5)(x-5)\right]\\ &= \log_b\left(x^2-25\right)\end{aligned}$$

Example 5
Rewriting Sums and Differences of Logarithms as Single Logarithms

Rewrite $\log_b\left(x^2-9\right)-\log_b(x-3)$ as a single logarithm.

Apply the property of logs that states $\log_b(M/N)=\log_b(M)-\log_b(N)$.

$$\begin{aligned}\log_b\left(x^2-9\right)-\log(x-3) &= \log_b\left[\frac{x^2-9}{x-3}\right]\\ &= \log_b\left[\frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}\right]\\ &= \log_b(x+3)\end{aligned}$$

Example 6
Estimating Logarithms

Given that $\ln(3)=1.1$, estimate the value of $\ln(250)$.

Find a power of three that approximates the given argument. Approximate the given logarithm using an approximate logarithm with a power of three as the argument.

$$\ln(250)\approx\ln(243)$$

Apply the property of logs that states $\log_b\left(M^p\right)=p\cdot\log_b(M)$.

$$\ln(250)\approx\ln(243)=\ln\left(3^5\right)=5\cdot\ln(3)\approx5\cdot1.1\approx5.5$$

Practice Set 3.7A

Evaluate the following logarithms. Round to the nearest thousandth when necessary.

#1 $\ln\left(e^3\right)$

#2 $\ln\left(e^3\right)+1$

#3 $\ln\left(\frac{1}{2}\right)$

#4 $\ln\left(\frac{1}{2}\right)+2$

#5 $\ln\left(\frac{1}{2}e\right)+\ln(2)$

#6 $\frac{10\ln(8)}{5\ln(2)}+1$

Simplify the following expressions. Do not round. Leave all answers exact.

#7 $2\ln(e)$

#8 $\ln(1)-\ln(e)$

#9 $3\ln(e)+5\ln(e)$

#10 $2\ln(3)$

#11 $\ln(1)-\ln(8)$

#12 $3\ln(2)+\ln(x)$

#13 $\frac{1}{3}\ln(x)$

#14 $\ln(x)-2\ln(y)$

#15 $3\ln(x)+\ln(x)$

#16 $x\ln(e)$

#17 $\ln(x)-2\ln(x)$

#18 $\frac{\log_2 2}{\log_2 e}$

<u>ANSWERS</u>

#1) 3
#2) 4
#3) –0.693
#4) 1.307
#5) 1
#6) 7
#7) 2
#8) –1
#9) 8
#10) ln(9)
#11) –ln(8)
#12) $\ln(8x)$
#13) $\ln\left(\sqrt[3]{x}\right)$
#14) $\ln\left(\frac{x}{y^2}\right)$
#15) $\ln(x^4)$
#16) x
#17) $\ln\left(\frac{1}{x}\right)$
#18) ln(2)

Practice Set 3.7B

Expand the following logarithms. Write each as a sum, difference or product of logarithms.

#1 $\log_b xy^2$

#2 $\ln\left(\frac{x+1}{y-7}\right)^2$

#3 $\log_2\left(\frac{rs^5}{t}\right)$

Write each sum, difference, or product of logarithms as a single logarithm.

#4 $\ln(x^2-3)+\ln(x)$

#5 $\log_b(12)-\log_b(6)$

#6 $\log_2(x^2-x-12)-\log_2(x-4)$

Use logarithm properties to evaluate *y*.

#7 $y=10^{\log 8}$

#8 $y=\ln e^{\frac{1}{4}}$

Evaluate the following logarithms given that $\ln(3)\approx 1.1$.

#9 ln(9)

#10 ln(27)

#11 ln(2,200)

#12 ln(6,600)

ANSWERS

#1) $\log_b x+2\log_b y$

#2) $2\ln(x+1)-2\ln(y-7)$

#3) $\log_2 r+5\log_2 s-\log_2 t$

#4) $\ln(x^3-3x)$

#5) $\log_b 2$

#6) $\log_2(x+3),\ x\neq 4$

#7) $y=8$

#8) $y=\frac{1}{4}$

#9) $\ln(9)=2\ln(3)\approx 2.2$

#10) $\ln(27)=3\ln(3)\approx 3.3$

#11) $\ln(2{,}200)\approx 7\ln(3)\approx 7.7$

#12) $\ln(6{,}600)\approx 8\ln(3)\approx 8.8$

Practice Set 3.7C

Expand the following logarithms. Write each as a sum, difference or product of logarithms.

#1 $\ln\left(\frac{1}{xy}\right)$

#2 $\log_b\left(\frac{x+2}{x-2}\right)$

#3 $\log_2\left(\frac{xy}{\sqrt{z}}\right)$

#4 $\log_2\left(x^2-3x-40\right)$

#5 $\log_4\left[\left(xy^2z^5\right)^2\right]$

Write each sum, difference, or product of logarithms as a single logarithm.

#6 $\ln\left(x^3+8\right)-\ln(x+2)$

#7 $\log_b(2)+\log_b(5)$

#8 $\log_2\left(x^2+2x-24\right)-\log_2(x+6)$

#9 $\frac{1}{3}\log_b(z)-\left[\log_b(x)+2\log_b(y)\right]$

#10 $\frac{1}{2}\log_b(121)+\log_b(3)$

<u>ANSWERS</u>

#1) $-\ln(x)-\ln(y)$

#2) $\log_b(x+2)-\log_b(x-2)$

#3) $\log_2(x)+\log_2(y)-\frac{1}{2}\log_2 z$

#4) $\log_2(x+5)+\log_2(x-8)$

#5) $2\log_4(x)+4\log_4(y)+10\log_4(z)$

#6) $\ln\left(x^2-2x+4\right),\ x\neq -2$

#7) $\log_b(10)$

#8) $\log_2(x-4),\ x\neq 6$

#9) $\log_b\left(\frac{\sqrt[3]{z}}{xy^2}\right)$

#10) $\log_b(33)$

Study Exercise 3.7

Problems

Write the sum of logarithms as a single logarithm.

#1 $\log_b(x+6)+\log_b(x)$

Write the difference of logarithms as a single logarithm.

#2 $\log_2(x^2-25)-\log_2(x+5)$

Expand the following logarithm. Write the logarithm as a sum or difference of logarithms without exponents in the arguments.

#3 $\log_3\left(\frac{x^4}{\sqrt{y}}\right)$

Recall the property: $\log_b(M)=\frac{\log_{b_2}(M)}{\log_{b_2}(b)}$. Use this property of logarithms to evaluate the following expression.

#4 $\frac{\log_B 64}{\log_B 4}$