Lecture 3.3

College Algebra

Instruction: ***Solving Exponential Equations without Logarithms***

This lecture uses a four-step process to solve exponential equations:

1 Isolate the base.
2 Write both sides of the equation as exponential expressions with like bases.
3 Set the exponents equal to each other.
4 Solve for the unknown.

Consider the following exponential equation: $\frac{1}{8}(2)^{2x+1} - 3 = 1$.

Following the rubric, the base should be isolated. In other words, algebraic transformations should be performed so that the expression 2^x stands alone on the left side of the equation as shown below.

$$\frac{1}{8}(2)^{2x+1} - 3 = 1$$
$$\frac{1}{8}(2)^{2x+1} = 1 + 3$$
$$\frac{1}{8}(2)^{2x+1} = 4$$
$$8 \cdot \left(\frac{1}{8}(2)^{2x+1} = 4\right)$$
$$(2)^{2x+1} = 32$$

Now that the base is isolated, the second step of the rubric requires that the right side of the equation be written as an exponential expression with the same base as the left side (in this case, 2).

$$(2)^{2x+1} = 32$$
$$(2)^{2x+1} = 2^5$$

The substitution above can be performed because $32 = 2^5$. Now, the third step of the rubric urges setting the two exponents, $2x + 1$ and 5, equal to one another. Logic maintains that the exponents must be equal if the bases are equal; thus,

$$2x + 1 = 5.$$

Finally, the fourth step of the rubric requires that the unknown be solved algebraically:

$$2x + 1 = 5$$
$$2x = 4$$
$$x = 2.$$

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Example 1
Solving an Exponential Equation

Solve the equation for *x*: $7\left(2^{x+3}\right)+3=3587$.

Isolate the base.

$$7\left(2^{x+3}\right)+3=3587$$
$$7\left(2^{x+3}\right)=3587-3$$
$$\frac{7\left(2^{x+3}\right)}{7}=\frac{3584}{7}$$
$$2^{x+3}=512$$

Recognize 512 as the ninth power of two.

$$2^{x+3}=2^9$$

Note that the two exponents must be equal.

If $2^{x+3}=2^9$, then $x+3=9$.

Solve the linear equation.

$$x+3=9$$
$$x=9-3$$
$$x=6$$

Example 2
Solving an Exponential Equation

Solve the equation for x: $5^{5x-12} = \dfrac{125^x}{5^x}$.

Note that 125 is the third power of 5.

$$5^{5x-12} = \frac{\left(5^3\right)^x}{5^x}$$

Recall that $\left(a^r\right)^s = a^{r \cdot s}$.

$$5^{5x-12} = \frac{5^{3x}}{5^x}$$

Recall that $\dfrac{a^r}{a^s} = a^{r-s}$ for all non-zero values of a.

$$5^{5x-12} = 5^{3x-x}$$
$$5^{5x-12} = 5^{2x}$$

Note that the two exponents must be equal.

$$\text{If } 5^{5x-12} = 5^{2x}, \text{ then } 5x - 12 = 2x.$$

Solve the linear equation.

$$5x - 12 = 2x$$
$$5x - 2x = 12$$
$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

Practice Set 3.3

Solve the following equations.

#1 $5^{2x+1} = 25$

#2 $27^{4x} = 9^{x+1}$

#3 $\left(\frac{1}{3}\right)^x = 81$

#4 $1.5^{x+1} = \left(\frac{27}{8}\right)^x$ Hint: $\left(\frac{3}{2}\right)^3 = \left(\frac{27}{8}\right)$

#5 $2^{2x+1} = \frac{1}{32}$

#6 $2^x = \frac{16}{2^x}$

#7 $16^x = 32$

#8 $1{,}000 = 0.1^x$

ANSWERS

#1 $x = \frac{1}{2}$
#2 $x = \frac{1}{5}$
#3 $x = -4$
#4 $x = \frac{1}{2}$
#5 $x = -3$
#6 $x = 2$
#7 $x = \frac{5}{4}$
#8 $x = -3$

Study Exercise 3.3

Problems

Solve the following equations for the exact value of the unknown.

#1 $4^x - 4 = 60$

#2 $3^{x+1} + 1 = 82$

#3 $2^{2x-1} = 1$

#4 $3(2)^{\frac{x}{2}} + 1 = 385$