College Algebra

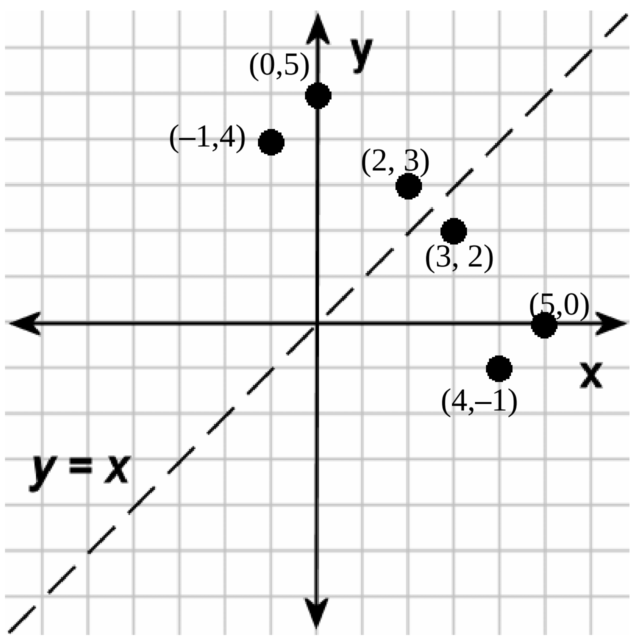
Instruction: *Inverse Functions*

Any one-to-one function has an inverse. A function is one-to-one if no two ordered pairs in the function have the same second component. In other words, one-to-one functions do not have any repeated values. Consequently, $q(x) = x^2$ is not one-to-one because $q(x) = 2$ when $x = -2$ and when $x = 2$, nor does $q(x)$ have an inverse (unless its domain is restricted). A quick test for one-to-one correspondence is the horizontal-line test. If all the horizontal lines of the Cartesian plane (the x-y plane) intersect the function only once, the function is one-to-one and has an inverse.

If a function is a one-to-one function, then its inverse is the function formed by reversing all the ordered pairs. In other words, for the function defined by the ordered pairs {(–1,4),(2,3),(5,0)} the inverse has the ordered pairs {(4, –1),(3,2),(0,5)}. If a function is labeled h, then its inverse is denoted h^{-1}.

Note the importance of the original function being one-to-one: if the original function is not one-to-one, then reversing the ordered pairs would yield a relation that is not a function.

Graphically, the points of h^{-1} are reflections of the points of h across the line $y = x$, as shown below.



Every one-to-one function has an inverse, and the graph of the inverse of any one-to-one function f can be found by reflecting the graph of f across the line $y = x$.

Instruction: *Inverse Functions*

Example 1
Finding the Inverse of an Exponential Function

Given $f(x) = 5 \cdot 3^{2x} + 1$, what is f^{-1}?

Rewrite *f*(*x*) as *y*.

$$y = 5 \cdot 3^{2x} + 1$$

Interchange *x* and *y*.

$$x = 5 \cdot 3^{2y} + 1$$

Solve for *y*.

$$x = 5 \cdot 3^{2y} + 1$$
$$x - 1 = 5 \cdot 3^{2y}$$
$$\frac{x-1}{5} = 3^{2y}$$
$$\log_3\left(\frac{x-1}{5}\right) = \log_3 3^{2y}$$
$$\log_3\left(\frac{x-1}{5}\right) = 2y$$
$$\frac{1}{2}\log_3\left(\frac{x-1}{5}\right) = y$$

Rewrite *y* using f^{-1} notation.

$$f^{-1}(x) = \frac{1}{2}\log_3\left(\frac{x-1}{5}\right)$$

Example 2
Finding the Inverse of an Logarithmic Function

Given $g(x) = \ln(x+3)+1$, what is g^{-1}?

Rewrite $g(x)$ as y.

$$y = \ln(x+3)+1$$

Interchange x and y.

$$x = \ln(y+3)+1$$

Solve for y.

$$x = \ln(y+3)+1$$
$$x-1 = \ln(y+3)$$
$$e^{x-1} = y+3$$
$$e^{x-1}-3 = y$$

Rewrite y using g^{-1} notation.

$$g^{-1}(x) = e^{x-1}-3$$

Practice Set 3.13

Find the inverses of the given functions. State the domain of the inverse function.

#1 $h(x) = -2^x + 3$

#2 $y(x) = e^x - 1$

#3 $g(x) = e^{2x}$

#4 $H(x) = \left(\frac{2}{3}\right)^x - \frac{4}{9}$

#5 $V(x) = 110{,}000(1.05)^{x/12}$

#6 $f(x) = -\log_3(x+3)$

ANSWERS

#1) $h^{-1}(x) = \log_2(-x+3)$ or $h^{-1}(x) = \frac{\ln(-x+3)}{\ln 2}$; domain $(-\infty,3)$

#2) $y^{-1}(x) = \ln(x+1)$; domain $(-1,\infty)$

#3) $g^{-1}(x) = \frac{1}{2}\ln x$ or $g^{-1}(x) = \ln\sqrt{x}$; domain $(0,\infty)$

#4) $H^{-1}(x) = \log_{2/3}\left(x + 4/9\right)$ or $H^{-1}(x) = \frac{\ln\left(x + 4/9\right)}{\ln\left(2/3\right)}$; domain $\left(-\frac{4}{9},\infty\right)$

#5) $V^{-1}(x) = \frac{12\ln\left(\frac{x}{110{,}000}\right)}{\ln(1.05)}$; domain $(0,\infty)$

#6) $f^{-1}(x) = 3^{-x} - 3$; domain $(-\infty,\infty)$

Study Exercise 3.13

Problems

Find the inverses of the given functions.

#1 $f(x) = e^{x-3}$

#2 $y(x) = \ln(x+2)$