Lecture 3.12

College Algebra

Instruction: *Graphing Exponential Functions with* X*-Intercepts*

Graphing exponential functions has been discussed both in *Lecture 3.A* and *Assignment 3.3*. This lecture focuses on finding the *x*-intercept of an exponential function. Consider the function:

$$f(x) = 2e^x - 1$$

Since *f*(*x*) has a shift down, the function has an *x*-intercept. Finding the *x*-intercept requires that the function be set equal to zero:

$$2e^x - 1 = 0$$

Solving this equation requires that the base be isolated:

$$2e^x - 1 = 0$$

$$2e^x = 1$$

$$e^x = \frac{1}{2}$$

Next, taking the natural log of both sides removes the variable from the exponent using the property of logs, $\log_b b^x = x$:

$$e^x = \frac{1}{2}$$

$$\ln e^x = \ln\frac{1}{2}$$

$$x = \ln\frac{1}{2}$$

$$x \approx -.69$$

Thus, *f*(*x*) crosses the *x*-axis at the point $\left(\ln\frac{1}{2}, 0\right)$.

For another example, consider the function:

$$g(x) = 2^x - .7$$

Since *g*(*x*) has a shift down, the function has an *x*-intercept. Finding the *x*-intercept requires that the function be set equal to zero:

$$2^x - .7 = 0$$

Solving this equation requires that the base be isolated:

$$2^x - .7 = 0$$

$$2^x = .7$$

Next, taking the natural log of both sides removes the variable from the exponent using the property of logs, $\log_b x^y = y \cdot \log_b x$:

$$2^x = .7$$

$$\ln 2^x = \ln .7$$

$$x \cdot \ln 2 = \ln .7$$

Isolating the variable solves the equation:

$$x \cdot \ln 2 = \ln .7$$

$$x = \frac{\ln .7}{\ln 2}$$

$$x \approx -.51$$

Thus, $g(x)$ crosses the x-axis at the point $\left(\frac{\ln 0.7}{\ln 2}, 0 \right)$.

Instruction: ***Graphing Exponential Functions with Vertical Translations***

Example 1
Graphing Exponential Functions with Vertical Translations

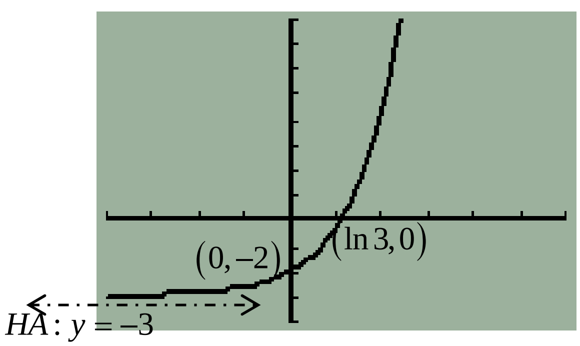
Graph $r(x) = e^x - 3$. Label intercepts and asymptotes. Show proper behavior.

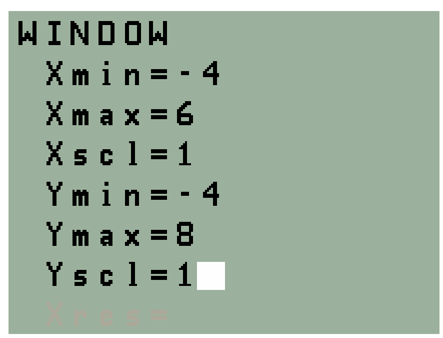
The initial amount (the coefficient to the base) is 1. The asymptotic value is negative three. The *y*-intercept is (0,–2).

Functions of the form $y(x) = ae^{kx}$ where $a > 0$ and $k > 0$ increase. Since $r(x)$ increases, it approaches zero as x approaches negative infinity, making the horizontal asymptote $y = -3$.

Set the function equal to zero and solve to find the *x*-value of the *x*-intercept.

$$e^x - 3 = 0$$
$$e^x = 3$$
$$\ln\left(e^x\right) = \ln(3)$$
$$x = \ln(3) \approx 1.1$$





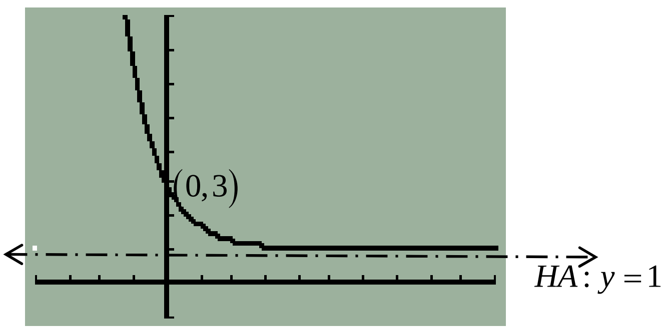
Example 2
Graphing Exponential Functions with Vertical Translations

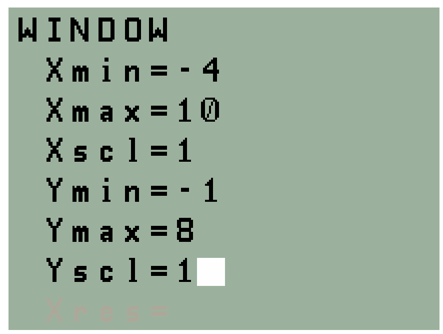
Graph $f(x) = 2e^{-x} + 1$. Label intercepts and asymptotes. Show proper behavior.

The initial amount (the coefficient to the base) is 2. The asymptotic value is positive one. The *y*-intercept is (0,3).

Functions of the form $y(x) = ae^{kx}$ where $a > 0$ and $k < 0$ decrease. Since *f*(*x*) decreases, it approaches zero as *x* approaches infinity, making the horizontal asymptote $y = 1$.

Since the asymptotic value is positive, the function does not have an *x*-intercept.

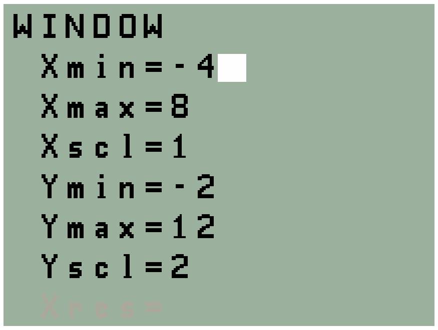


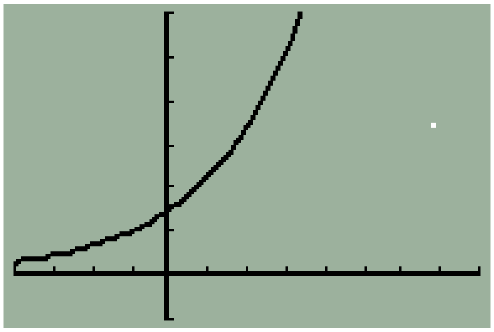


Example 2
Graphing Exponential Functions

Graph $q(x) = 3(1.5)^x$. Label intercepts and asymptotes. Show proper behavior.

The initial amount (the coefficient to the base) is 3. The *y*-intercept is (0,3). The asymptotic value is zero. The common ratio is 1.5. Since the common ratio is greater than one, the function increases and approaches zero as *x* approaches negative infinity, making the horizontal asymptote *y* = 0 (the *x*-axis).





X	Y1
-2	1.3333
-1	2
0	3
1	4.5
2	6.75
3	10.125
4	15.188

Y1=3(1.5)^X

Example 3
Graphing Exponential Functions

Graph $q(x) = 10(0.5)^x$. Label intercepts and asymptotes. Show proper behavior.

The initial amount (the coefficient to the base) is 10. The *y*-intercept is (0,10). The asymptotic value is zero. The common ratio is 0.5. Since the common ratio is less than one, the function decreases and approaches zero as *x* approaches infinity, making the horizontal asymptote *y* = 0 (the *x*-axis).

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WINDOW
  Xmin=-3
  Xmax=6
  Xscl=1
  Ymin=-4
  Ymax=16
  Yscl=4
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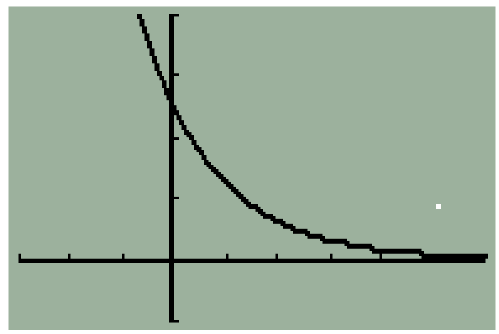


X	Y1	
-2	40	
-1	20	
0	10	
1	5	
2	2.5	
3	1.25	
4	.625	

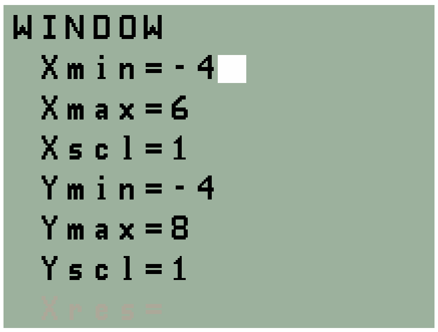
Y1=10(0.5)^X

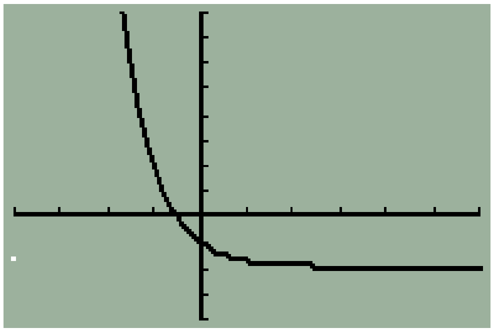
Example 4
Graphing Exponential Functions

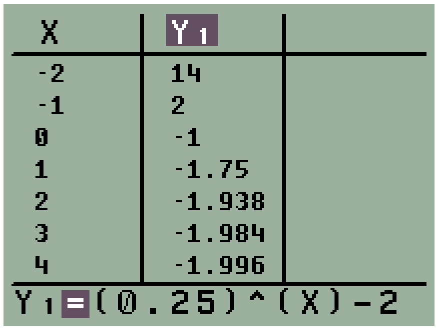
Graph $g(x) = \left(\frac{1}{4}\right)^x - 2$. Label intercepts and asymptotes. Show proper behavior.

The initial amount (the coefficient to the base) is 1. The asymptotic value is negative two. Since 1 + –2 = –1, the *y*-intercept is (0, –1). The common ratio is $1/4$. Since the common ratio is less than one, the function decreases and approaches negative two (the asymptotic value) as *x* approaches infinity, making the horizontal asymptote *y* = –2. Setting the equation equal to zero and solving yields the *x*-value of the *x*-intercept.

$$\left(\frac{1}{4}\right)^x - 2 = 0$$
$$(4)^{-x} = 2$$
$$(4)^{-x} = \sqrt{4}$$
$$(4)^{-x} = 4^{\frac{1}{2}}$$
$$-x = 1/2$$
$$x = -1/2$$
$$\therefore$$
$$(-1/2, 0)$$







X	Y_1
-2	14
-1	2
0	-1
1	-1.75
2	-1.938
3	-1.984
4	-1.996

Y_1=(0.25)^(X)-2

Practice Set 3.12

For each of the following functions, state whether it increases or decreases, name the *x* and *y*-intercepts, and identify the horizontal asymptote.

#1 $h(x) = 2^x - 1$

#2 $y(x) = 25 \cdot e^x - 40$

#3 $g(x) = 20e^{5x} + 40$

#4 $H(x) = \left(\frac{2}{3}\right)^x - \frac{4}{9}$

#5 $Y(x) = -20 \cdot e^x + 40$

#6 $f(x) = 2(1.5)^x - 5$

ANSWERS

#1) $h(x)$ increases, the *x* & *y*-intercept occurs at (0,0), and its horizontal asymptote is $y = -1$.

#2) $y(x)$ increases, the *x*-intercept is $(\ln{}^{8}/_{5}, 0)$, the *y*-intercept is (0, –15), and its horizontal asymptote is $y = -40$.

#3) $g(x)$ increases, $g(x)$ does not have an *x*-intercept, the *y*-intercept occurs at (0, 60), and its horizontal asymptote is $y = 40$.

#4) $H(x)$ decreases, *x*-intercept occurs at (2,0), the *y*-intercept occurs at $(0, {}^{5}/_{9})$, and its horizontal asymptote is $y = -{}^{4}/_{9}$.

#5) $Y(x)$ decreases, *x*-intercept occurs at (ln2,0), the *y*-intercept occurs at (0, 20), and its horizontal asymptote is $y = 40$.

#6) $f(x)$ increases, *x*-intercept occurs at $({}^{\ln 2.5}/_{\ln 1.5}, 0)$, the *y*-intercept occurs at (0, –3), and its horizontal asymptote is $y = -5$.

Problems

Find the *x*-intercepts of the following exponential functions. Recall that an *x*-intercept is a point on the graph, i.e., an ordered pair. Write irrational numbers in simplified exact terms.

#1 $h(x) = 2^x - 3$

#2 $y(x) = e^{-0.1x} - 3$

Lecture 3.14

College Algebra

Instruction*: Graphing Logarithmic Functions*

The rubric below represents an effective method for graphing logarithmic functions.

1. Find the domain by finding where the argument of the log is positive.

2. Recognize the vertical asymptote. The vertical asymptote is generally $x = 0$. Often it will be the line represented by the equation x = the left boundary of the domain. If the function has been reflected across the y-axis, the vertical asymptote will be the line represented by the equation x = the right boundary of the domain.

3. Find the x-intercept by setting the function equal to zero.

4. Find the y-intercept (if there is one) by substituting zero for x.

5. Find some general values for the function substituting values for x. Be sure to use some fractional values between the x-intercept and the two integers just below and just above it.

The example on the following page uses the above rubric to graph a logarithmic function.

Lecture 3.14

Example: $f(x) = \log_2(x+2)$

Step 1: $x + 2 > 0$

$x > -2$

$(-2, \infty)$

Find the domain. The domain of a logarithmic function will be limited to where the argument is positive. Solving for where the argument is greater than zero will find the domain.

Step 2: vertical asymptote: $x = -2$

If the function is not reflected across the y-axis, the vertical asymptote will be represented by the equation x = the left boundary of the domain.

Step 3:

$$\log_2(x+2) = 0$$

$$2^{\log_2(x+2)} = 2^0$$

$$(x+2) = 1$$

$$x = 1 - 2$$

$$x = -1$$

$$(-1, 0)$$

To find the x-intercept, set the function equal to zero and solve for x.

Step 4:

$$f(x) = \log_2(x+2)$$

$$f(0) = \log_2(0+2)$$

$$f(0) = \log_2 2$$

$$f(0) = 1$$

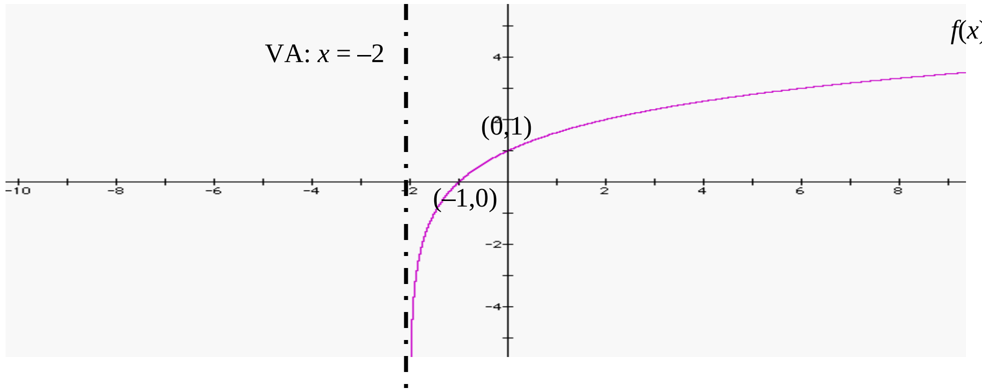
$$(0, 1)$$

To find the y-intercept, substitute zero for x and evaluate.

Step 5:

x	$f(x)$
–1.75	–2
–1.5	–1
–1.25	–.415
–1	0
–.75	.322
–.5	.585
0	1
1	1.585
2	2

To find general values of the function, substitute some values for x into the function and evaluate. To evaluate while using a graphing calculator, use the change of base formula. For example, to evaluate $y = \log_2(-1.25 + 2)$, change the base to e, and type: $\frac{\ln(-1.25+2)}{\ln(2)}$. To use the graphing utility, hit the "Y=" key then type the above. After entering the function, hit the TBLSET key (using the "2ND" key). Start the table at the beginning of the domain (in this case –2), and set the table to change ("ΔTbl") at some fractional value such as .25. After setting these table characteristics, hit the TABLE key (using the "2ND" key). A table much like the one to the left will appear. Use the arrow keys to scroll up and down.



Instruction: ***Graphing Logarithmic Functions***

Example 1
Graphing Logarithmic Functions

Graph $f(x) = \log_2(x-1)$. Label intercepts and asymptotes and show proper behavior.

Determine the domain. Recall that the argument of a logarithm must be a positive number.

$$x - 1 > 0$$
$$x > 1$$

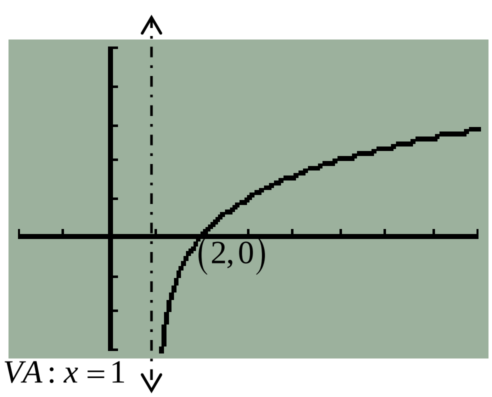
Identify the asymptote. Logarithmic functions of the form $y(x) = \log_b(x+a)$ have vertical asymptotes whose equation takes the form $x = \text{boundary on the domain}$.

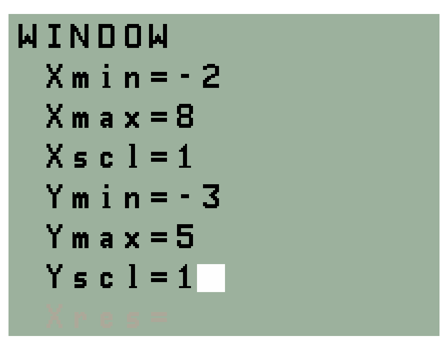
$$VA: x = 1$$

Identify the intercepts. Logarithmic functions of the form $y(x) = \log_b(x)$ only have *y*-intercepts if shifted to the left. Since *f*(*x*) is shifted to the right, *f*(*x*) does not have a *y*-intercept. To find the *x*-value of the *x*-intercept, set the function equal to zero and solve, i.e., find the roots of the function.

$$\log_2(x-1) = 0$$
$$x - 1 = 2^0$$
$$x - 1 = 1$$
$$x = 2$$
$$\therefore$$
$$(2, 0)$$

Determine if the function decreases or increases. Functions of the form $y(x) = \log_b(x)$ increase if b > 1 (without reflection).





Example 2
Graphing Logarithmic Functions

Graph $h(x) = \ln(x+3)$. Label intercepts and asymptotes and show proper behavior.

Determine the domain. Recall that the argument of a logarithm must be a positive number.

$$x + 3 > 0$$
$$x > -3$$

Identify the asymptote. Logarithmic functions of the form $y(x) = \log_b(x+a)$ have vertical asymptotes whose equation takes the form $x = \text{boundary on the domain}$.

$$VA: x = -3$$

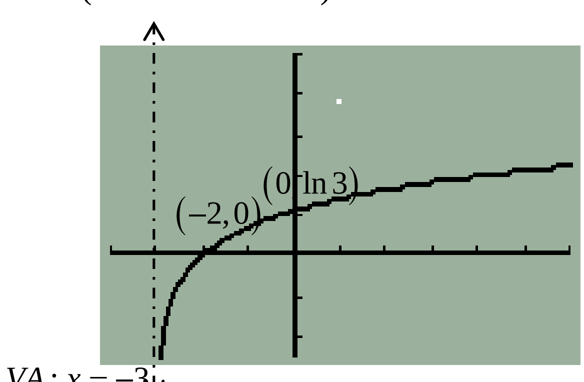
Identify the intercepts. To find the y-intercept, evaluate *h*(0).

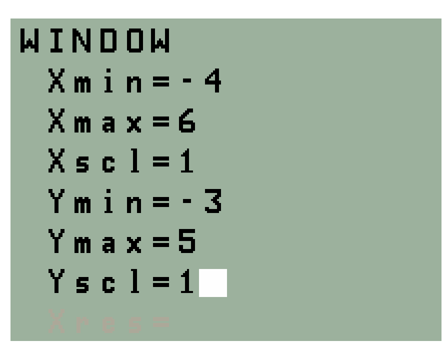
$$h(0) = \ln(0+3)$$
$$h(0) = \ln(3) \approx 1.1$$

To find the *x*-value of the *x*-intercept, set the function equal to zero and solve, i.e., find the roots of the function.

$$\ln(x+3) = 0$$
$$x + 3 = e^0$$
$$x + 3 = 1$$
$$x = -2$$
$$\therefore$$
$$(-2, 0)$$

Determine if the function decreases or increases. Functions of the form $y(x) = \log_b(x)$ increase if b > 1 (without reflection).





Practice Set 3.14

Graph the following logarithmic functions. Label all intercepts and asymptotes. Indicate the domain of each function. Draw graphs neatly with evenly spaced units along each axis.

#1 $y(x) = 2\ln x$

#2 $f(x) = \log_5 x$

#3 $g(x) = \log_5(-x)$

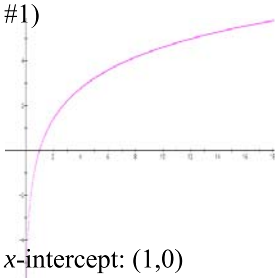
#4 $h(x) = \log_{\frac{1}{3}}(x)$

#5 $k(x) = \log_4(x)$

#6 $q(x) = -1 + \log_4(x+3)$

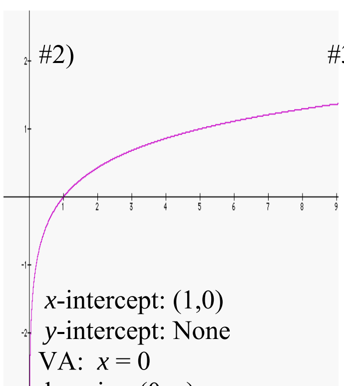
<u>ANSWERS</u>

#1)



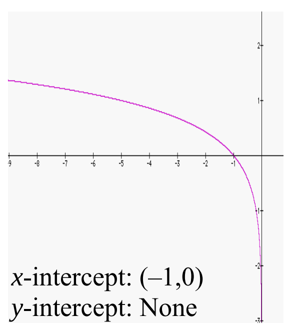
x-intercept: (1,0)
y-intercept: None
VA: $x = 0$
domain: $(0,\infty)$

#2)



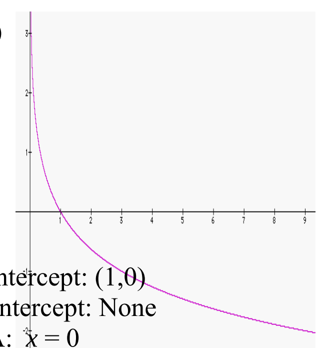
x-intercept: (1,0)
y-intercept: None
VA: $x = 0$
domain: $(0,\infty)$

#3)



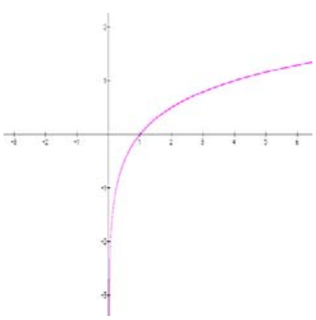
x-intercept: (–1,0)
y-intercept: None
VA: $x = 0$
domain: $(-\infty,0)$

#4)



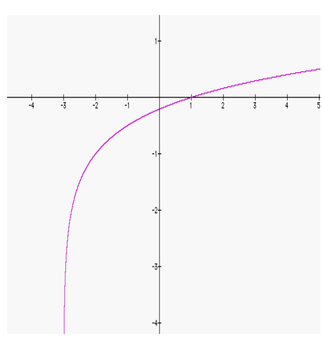
x-intercept: (1,0)
y-intercept: None
VA: $x = 0$
domain: $(0,\infty)$

#5)



x-intercept: (1,0)
y-intercept: None
VA: $x = 0$
domain: $(0,\infty)$

#6



x-intercept: (1,0)
y-intercept: $(0,-1+\log_4 3)$
VA: $x = -3$
domain: $(-3,\infty)$

Problems

Graph the following logarithmic function. Label both intercepts and the asymptote. Indicate the domain of the function in the space provided.

#1 $y(x) = \ln(x+2)$ Domain of $y(x)$:______________________________