Lecture 2.8

College Algebra

Instruction: *Graphing Polynomial Functions*

PROBLEM:

Graph $P(x) = 3x^4 + 5x^3 - 64x^2 - 74x - 20$. Label all intercepts.

SOLUTION:

Determine that there is one positive real root and three (or one) negative real root using Descartes' Rule of Signs. Determine also that the possible rational roots are ±1, ±2, ±4, ±5, ±10, ±20, $\pm\frac{1}{3}$, $\pm\frac{2}{3}$, $\pm\frac{10}{3}$, $\pm\frac{20}{3}$.

Use synthetic division to find the rational roots. Use numbers from the list above. Divide until getting zero as a remainder.

-5	3	5	-64	-74	-20
		-15	50	70	20
	3	-10	-14	-4	0

Synthetic division with –5 as the divisor reveals that –5 is a root and that $3x^3 - 10x^2 - 14x - 4$ is a cubic factor of the polynomial. Divide into $3x^3 - 10x^2 - 14x - 4$ next in order to reduce the polynomial to a quadratic factor.

$-\frac{2}{3}$	3	-10	-14	-4
		-2	8	4
	3	-12	-6	0

$-\frac{2}{3}$ is a root, and $3x^2 - 12x - 6$ is a quadratic factor of the polynomial. To find the remaining two roots, set the quadratic equal to zero and solve.

$$3x^2 - 12x - 6 = 0$$

There is a greatest common factor so reduce the equation.

$$\frac{3x^2}{3} - \frac{12x}{3} - \frac{6}{3} = \frac{0}{3}$$

$$x^2 - 4x - 2 = 0$$

Use the quadratic equation to solve as shown on the next page.

$$x^2 - 4x - 2 = 0$$

$a = 1$, $b = -4$, and $c = -2$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16+8}}{2}$$

$$x = \frac{4 \pm \sqrt{24}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \cdot 6}}{2}$$

$$x = \frac{4 \pm 2\sqrt{6}}{2}$$

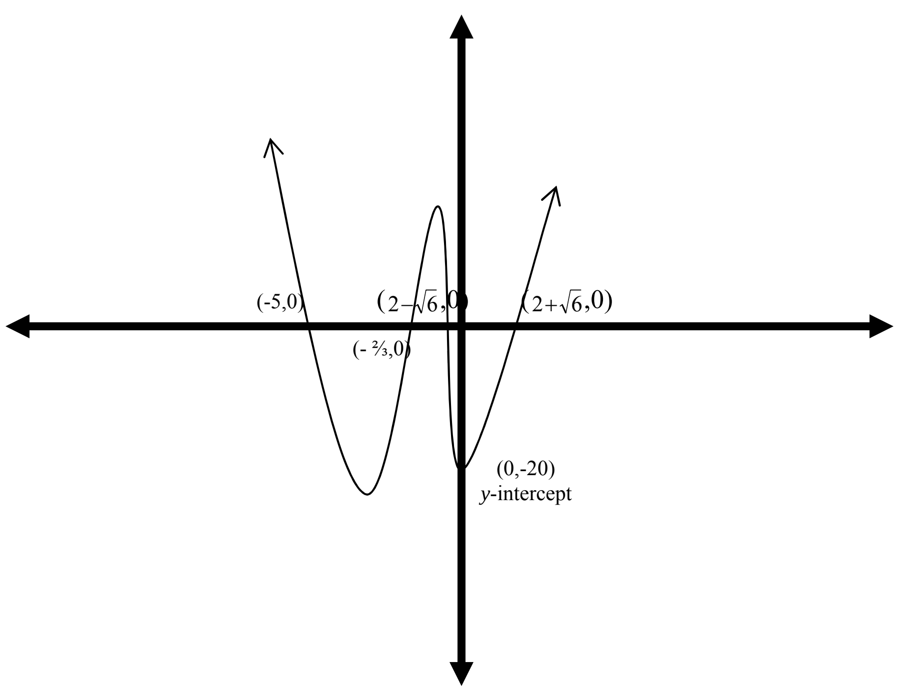
$$x = \frac{4}{2} \pm \frac{2\sqrt{6}}{2}$$

$$x = 2 \pm \sqrt{6}$$

Now that all the roots are found, we see that they are all real & unique roots, meaning there are four *x*-intercepts. One on the positive side of the *x*-axis at $x = 2 + \sqrt{6}$, and three on the negative side of the *x*-axis at $x = -5$, $x = -\frac{2}{3}$, and $x = 2 - \sqrt{6}$.

The *y*-intercept is always at the constant, so this polynomial has a *y*-intercept at (0, –20).

The right end is going up because the leading coefficient is positive. The left end is also going up because the degree is even so the polynomial has same end behavior.



Instruction: ***Graphing Polynomial Functions***

Example 1
Graphing Polynomial Functions

Graph $f(x) = 2x^4 + 15x^3 + 30x^2 + 20x + 3$. Label all intercepts. Show proper end behavior.

According to Descartes' Rule of Signs, *f*(*x*) has no positive roots because it has no sign changes in its expanded form. Divide synthetically by possible rational roots until a zero appears in the last entry of the quotient row.

$$\begin{array}{r|rrrrr} -1 & 2 & 15 & 30 & 20 & 3 \\ & & -2 & -13 & -17 & -3 \\ \hline & 2 & 13 & 17 & 3 & 0 \end{array}$$

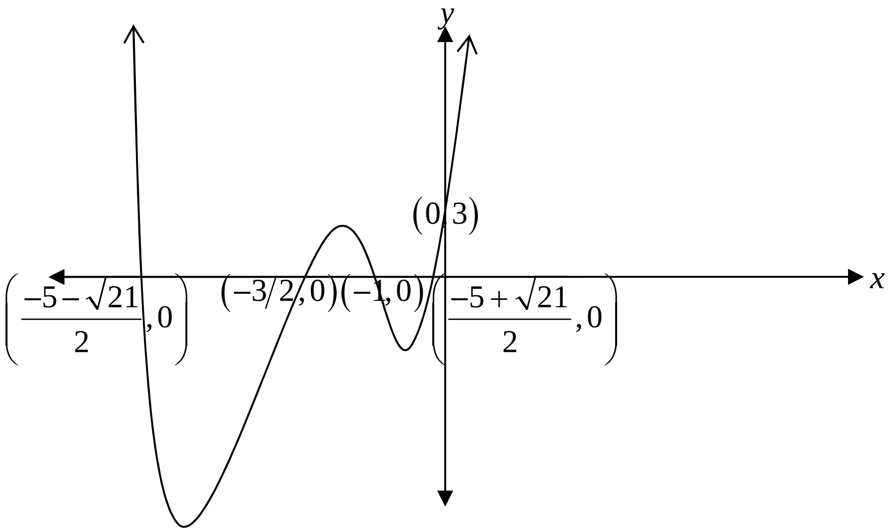
The zero indicates that the divisor, –1, is a root. Interpret the quotient row as a cubic factor of the polynomial $2x^3 + 13x^2 + 17x + 3$. Use the cubic factor as the dividend and synthetically divide until a zero appears in the last entry of the quotient row.

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 13 & 17 & 3 \\ & & -3 & -15 & -3 \\ \hline & 2 & 10 & 2 & 0 \end{array}$$

The zero remainder indicates that the divisor, $-3/2$, is a root. Interpret the quotient row as a quadratic factor of the polynomial $2x^2 + 10x + 2 = 2(x^2 + 5x + 1)$. Set the quadratic factor equal to zero and solve.

$$x^2 + 5x + 1 = 0$$
$$x = \frac{-5 \pm \sqrt{25-4}}{2} = \frac{-5 \pm \sqrt{21}}{2}$$

Note that both ends will rise (because the degree is even). Graph the polynomial. Label the intercepts.



Example 2
Graphing Polynomial Functions

Graph $y(x) = -2x^5 - 18x^4 - 54x^3 - 54x^2$. Label all intercepts. Show proper end behavior.

Note that $y(x)$ has a zero constant. Factor the greatest common factor.

$$y(x) = -2x^5 - 18x^4 - 54x^3 - 54x^2$$
$$y(x) = -2x^2\left(x^3 + 9x^2 + 27x + 27\right)$$

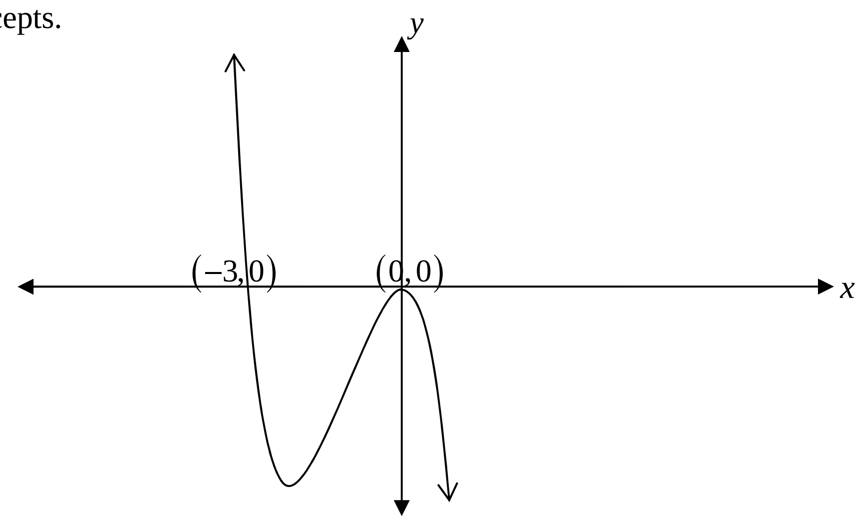
Note that zero is a double root (a root with multiplicity of two) of $y(x)$. Find the remaining roots by finding the roots of the cubic factor.

According to Descartes' Rule of Signs, the cubic has no positive roots because it has no sign changes in its expanded form. Divide synthetically by negative possible rational roots until a zero appears in the last entry of the quotient row.

$$\begin{array}{r|rrrr} -3 & 1 & 9 & 27 & 27 \\ & & -3 & -18 & -27 \\ \hline & 1 & 6 & 9 & 0 \end{array}$$

The zero remainder indicates that the divisor, –3, is a root. Interpret the quotient row as a quadratic factor of the polynomial $x^2 + 6x + 9$. Factor $x^2 + 6x + 9$ to $(x+3)(x+3)$. These two factors plus the factor implied by the "successful" synthetic division above indicate that –3 is a triple root (a root with multiplicity of three, meaning a root whose linear factor is repeated three times).

Note that the ends will have opposite behavior because the degree is odd. Note that the polynomial falls on the right because the leading coefficient is negative. Note that the graph will "bounce off" the x-axis at zero since the multiplicity of zero is even. Graph the polynomial. Label the intercepts.



Practice Set 2.8

Graph the following polynomial functions. Label all intercepts. Show appropriate end behavior. Show appropriate behavior near the *x*-axis.

#1 $f(x) = x^3 - 3x^2$

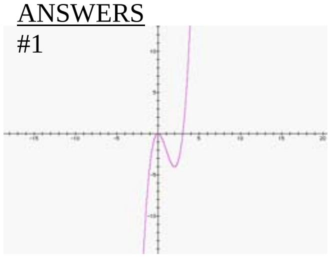
#2 $F(x) = \frac{1}{4}x^4 + \frac{3}{4}x^3 - \frac{9}{4}x^2 - \frac{23}{4}x - 3$

#3 $g(x) = -x^3 + 3x^2 - 4$

#4 $p(x) = \frac{1}{8}x\left(x^4 - 2x^3 - 10x^2 + 18x + 9\right)$

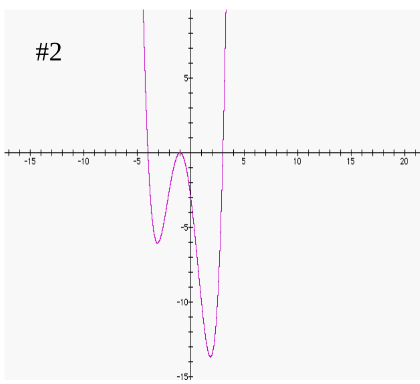
<u>ANSWERS</u>

#1



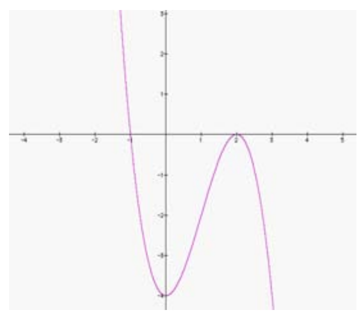
x-intercepts (0,0) & (3,0)
y-intercept (0,0)

#2



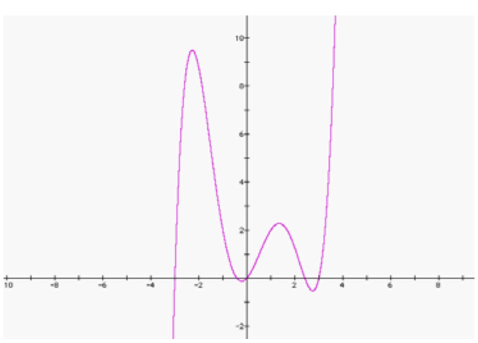
x-intercepts (–4,0), (–1,0) & (3,0)
y-intercept (0,–3)

#3



x-intercepts (–1,0) & (2,0)
y-intercept (0, –4)

#4



x-intercepts (–3,0), $\left(1-\sqrt{2},0\right)$, (0,0), $\left(1+\sqrt{2},0\right)$, & (3,0)
y-intercept (0,0)

Practice Set 2.8_Supplemental

Graph the following polynomial functions. Label all intercepts. Show appropriate end behavior. Show appropriate behavior near the *x*-axis.

#1 $p(x)=2x^4+11x^3+13x^2-11x-15$

#2 $f(x)=x^5+2x^4-x^3-2x^2$

#3 $g(x)=-x^4+3x^3-5x^2+x+10$

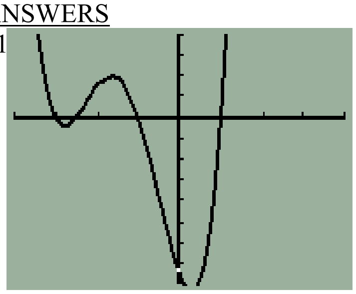
#4 $h(x)=12x^3-29x^2+10x$

#5 $\lambda(x)=-2x^4+x^3+24x^2+20x-16$

#6 $\beta(x)=x^4+x^3-12x^2+2x+8$

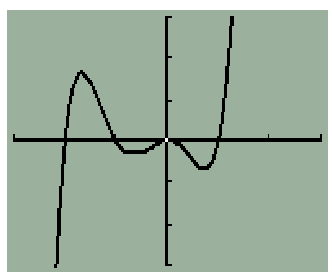
ANSWERS

#1



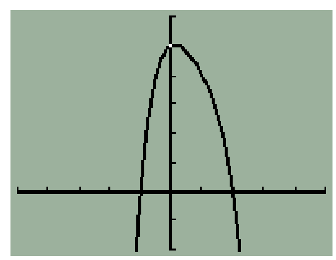
unit on *x*-axis: 1
unit on *y*-axis: 2
x-int: (–3,0), $(-{}^5/_2,0)$,(–1,0),(1,0)
y-int: (0,–15)
roots: –3, $-{}^5/_2$, –1, 1

#2



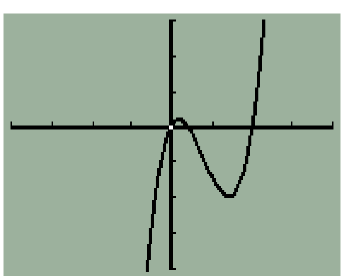
unit on *x*-axis: 1
unit on *y*-axis: 1
x-int: (–2,0), (–1,0),(0,0),(1,0)
y-int: (0,0)
roots: –2, –1, 0 (M2), 1

#3



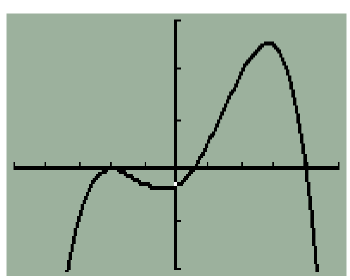
unit on *x*-axis: 1
unit on *y*-axis: 2
x-int: (–1,0) & (2,0)
y-int: (0,10)
roots: –1, 2, 1 + 2*i*, 1 – 2*i*

#4



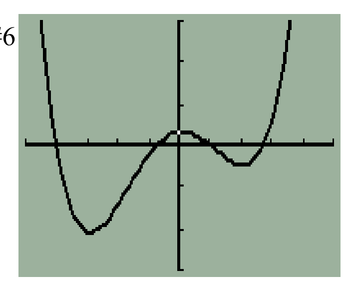
unit on *x*-axis: 1
unit on *y*-axis: 5
x-int: (0,0),$({}^5/_{12},0)$,(2,0)
y-int: (0,0)
roots: 0, ${}^5/_{12}$, 2

#5



unit on *x*-axis: 1
unit on *y*-axis: 50
x-int: (–2,0),(½,0),(4,0)
y-int: (0,–16)
roots: –2 (M2), ½, 4

#6



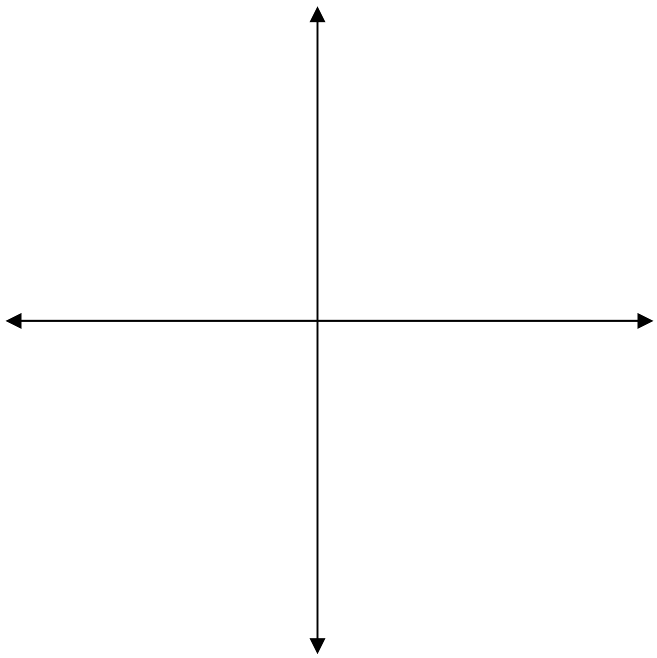
unit on *x*-axis: 1
unit on *y*-axis: 25
x-int: (–4,0),(1,0),$(1+\sqrt{3},0)$, $(1-\sqrt{3},0)$
y-int: (0,8)
roots: –4, 1, $1+\sqrt{3}$, $1-\sqrt{3}$

Study Exercise 2.8

Problems

Sketch the graphs of the following polynomial functions. Label the *x*-intercepts and *y*-intercept of each graph. Indicate correct end behavior as well as the correct behavior near the *x*-axis according to the multiplicity of the roots.

#1 $y(x) = 2x^3 - 9x^2 - 15x + 50$



#2 $p\left(x\right) = x^4 - 3x^3 - 10x^2 + 18x + 24$