Lecture 1.5

College Algebra

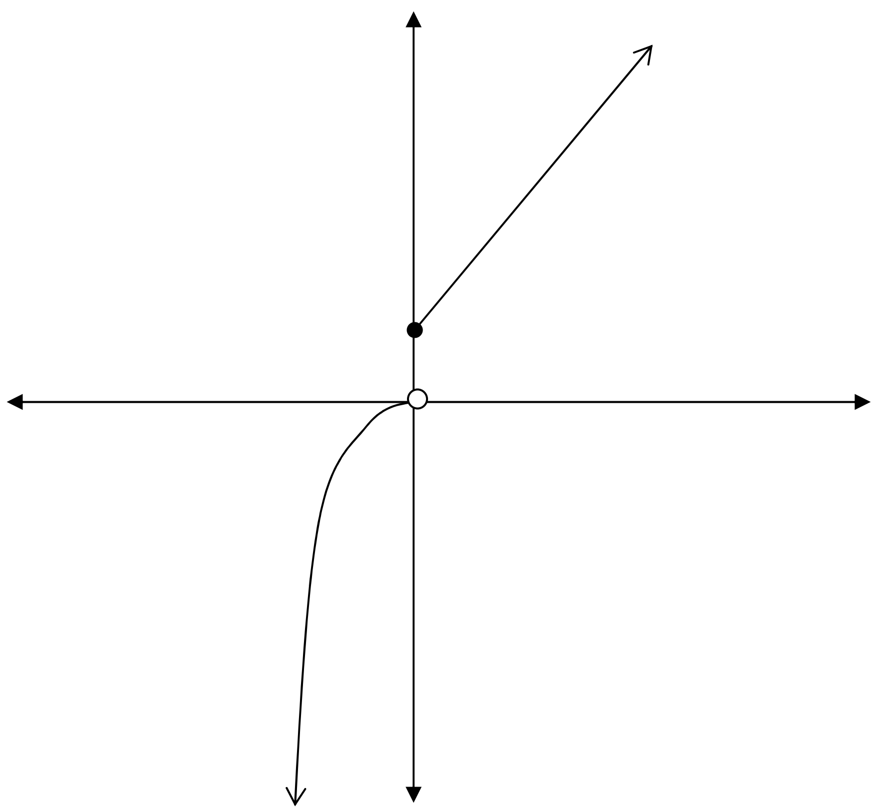
Instruction: *Piecewise Functions*

$$p(x)=\begin{cases} x^3 & if \;\; x<0 \\ \\ x+1 & if \;\; x\geq 0. \end{cases}$$

Piecewise functions are functions whose definition involves more than one formula. For example, consider $p(x)$ above.

Since $p(x)$ is defined by two formulas [$p(x) = x^3$ for x-values below zero and $p(x) = x + 1$ for x-values greater than or equal to zero], it is a piecewise function. When generating its table of ordered pairs, substitute values less then zero into x^3 and values equal to or greater than zero into $x + 1$. Since the first piece of the function has values corresponding to x-values that approach zero, it may be helpful to substitute zero for x in the first formula just to determine an end point of the first piece. The graph should have an open circle at this end-point unless it corresponds to the beginning point of the next piece of the function.

x	$p(x) = x^3$	$p(x) = x + 1$
-2	-8	
-1	-1	
0	0 (not a value of the function)	
0		1
1		2
2		3



An Informal Discussion continued. . .

In Sections 1.1-1.3, we discussed the monthly income of a paperboy who earns \$4.50 for every subscriber to whom he delivers the paper. We used the function below to describe the paperboy's monthly income.

$$p(x) = \$4.50x$$

Sometimes, real-world functions are much more complicated. For instance, let's consider the paperboy's weekly income (instead of his monthly income) assuming that he gets paid per hour of work (instead of per subscriber serviced). In the real-world, the paperboy would earn more per hour if he worked more than forty hours per week. Let's assume he averages \$6.00 per hour if he works more than forty hours in the week and that he cannot work more than eighty hours according to company policy. According to these assumptions, we can represent the paperboy's weekly income with a piece-wise function given below.

$$P(x) = \begin{cases} \$4.50x & \text{if } 0 \le x \le 40 \\ \$6.00x & \text{if } 40 < x \le 80 \end{cases}$$

Recall that a function only assigns one y-value to any given x-value. Accordingly, when we try to evaluate $P(20)$ and $P(50)$, we must ask ourselves, "Which rule must be used?" In this case, we either multiply the x-value by \$4.50 or by \$6.00. For $P(20)$ where the input value is twenty, we multiply by \$4.50 because that is the rule for x-values from zero to forty. For $P(50)$ where the input value is fifty, we multiply by \$6.00 because that is the rule for x-values greater than forty but less than or equal to eighty. Thus, $P(20) = \$90.00$ and $P(50) = \$300.00$.

We might also note that our piece-wise function has a stated domain. The "if statements" represent "pieces" of the domain, and all the "if statements" taken together give the full domain. The domain of $P(x)$ in our discussion includes all the real numbers from zero to eighty, written in set interval notation as [0,80].

Instruction: *Piece-wise Functions*

Example 1
Evaluating Piece-wise Functions

Consider $f(x)=\begin{cases}-x-1 & if\ \ x<0\\ 2x & if\ \ x\geq 0\end{cases}$. Which is greater $f(1/2)$ or $f(-3)$?

Evaluate $f(1/2)$. Note that $1/2>0$, and use the "piece" for *x*-values greater than zero.

$$f(1/2)=2(1/2)=1$$

Evaluate $f(-3)$. Note that $-3<0$, and use the "piece" for the *x*-values less than zero.

$$f(-3)=-(-3)-1=3-1=2$$

Since $f(1/2)=1$ and $f(-3)=2$, $f(-3)>f(1/2)$.

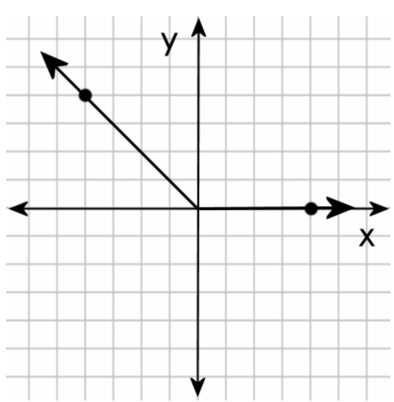
Example 2
Graphing Piece-wise Functions

Consider $P(x)=\begin{cases}-x & if\ \ x<0\\ 0 & if\ \ x\geq 0\end{cases}$. Sketch the graph of $P(x)$.

The function $P(x)$ has two "pieces" so-to-speak, meaning it has two rules. One rule states that the function's range value is the opposite of any *x*-value less than zero. For instance if $x = –4$, then $y = 4$. If $x = –3$, then $y = 3$. If $x = –2$, then $y = 2$ and so on until the *x*-values reach zero where the second rule applies.

For *x*-values equal to zero or greater, the rule states that the function's range value is zero for any *x*-value greater than or equal to zero. For instance, if $x = 0$, then $y = 0$, and if $x = 4$, then $y = 0$.

The graph of the "piece" for *x*-values less than zero is linear. The function decreases by one for every single-unit increase in *x* (i.e., it has a slope of negative one). The graph of the "piece" for *x*-values equal to or greater than zero is constant, which is a horizontal line along the *x*-axis.



Practice Set 1.5

Graph the given piecewise function.

#1 $p(x) = \begin{cases} x^2 & \text{if } -2 \le x \le 2 \\ 4 & \text{if } x < -2 \text{ or } x > 2 \end{cases}$

#2 $d(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x \ge 0 \end{cases}$

#3 $P(x) = \begin{cases} x+1 & \text{if } x \le 2 \\ x-1 & \text{if } x > 2 \end{cases}$

#4 $f(x) = \begin{cases} x-1 & \text{if } x \le 2 \\ x+1 & \text{if } x > 2 \end{cases}$

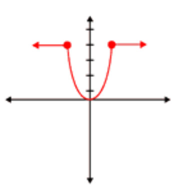
#5 $g(x) = \begin{cases} (x+4)^2 - 2 & \text{if } x \le -2 \\ -x & \text{if } -2 < x \le 2 \\ -(x-4)^2 + 2 & \text{if } x > 2 \end{cases}$

#6 $y(x) = \begin{cases} x^2 - 2 & \text{if } x < 1 \\ -2x + 4 & \text{if } x \ge 1 \end{cases}$

#7 $h(x) = \begin{cases} 3 - x & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases}$

#8 $S(x) = \begin{cases} 1 & \text{if } 0 < x \le 1 \\ 2 & \text{if } 1 < x \le 2 \\ 3 & \text{if } 2 < x \le 3 \\ & \vdots \\ n & n-1 < x \le n \end{cases}$

#1



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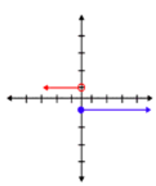

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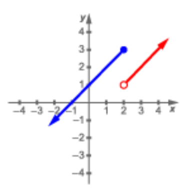

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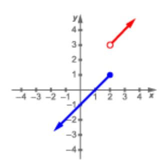

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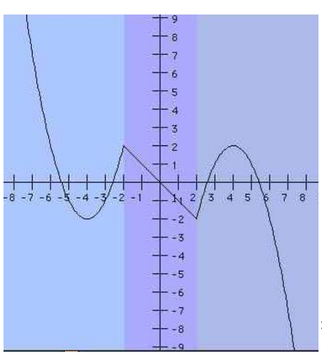

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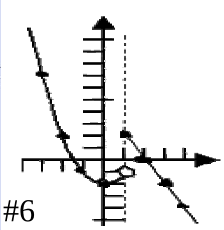

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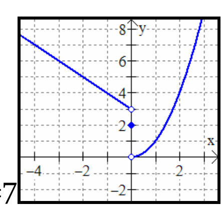

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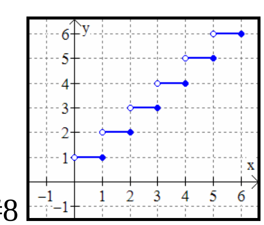

Study Exercise 1.5

Problems

Consider functions *f* and *g*.

$$f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases} \qquad g(x) = \begin{cases} x^3 & \text{if } x < 0 \\ -x^2+1 & \text{if } x \geq 0 \end{cases}$$

#1 Evaluate $f(5)$

#2 Evaluate $g(-2)$.

#3 Evaluate $g(0)$

#4 Sketch the graph of $f(x)$.