College Algebra

Lecture: *Polynomial Functions, Terminology*

POLYNOMIAL FUNCTIONS

Polynomial Functions – Functions of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ where the coefficients, $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$, are real number coefficients and n is a positive integer called the degree.

Examples of polynomial functions:

These examples are discussed below.

$f(x) = x^5 + 4x^4 - 11x^3 - 16x^2 + 58x - 60$
$g(x) = 5 - x^2$
$h(x) = 4x^2 + 12x + 9$
$d(t) = -t^2 - 9$
$j(k) = 2(k + 1)^2(k - 3)^3$
$v(x) = x^3 - 8$
$w(x) = x^2 + 6x + 8$

INTRODUCTORY TERMINOLOGY

Real Numbers – Real numbers include all rational and irrational numbers.

Rational Numbers – rational numbers are numbers that can be written in the form $^a/_b$, where a and b are integers and $b \neq 0$ (i.e., the fractions). In their decimal form, rational numbers repeat or terminate. For example, $^1/_3$ can be written $.\overline{3}$ with the three repeating, and ½ can be written .5 terminating in the tenths place. Rational numbers include numbers like 1, –2, 15, ½, $^2/_3$, and $-5\,^1/_3$.

Irrational Numbers – irrational numbers are numbers that are not rational, i.e., with the above definition, irrational numbers do *not* terminate or repeat in their decimal form. Irrational numbers include numbers like $\sqrt{2}, 1+2\sqrt{3}, 7-\sqrt{5}$, and π.

Complex Numbers – complex numbers are numbers of the form $a + bi$ where a and b are real numbers, and
$i = \sqrt{-1}$. In other words, complex numbers include numbers with imaginary parts. Complex numbers originate from taking the square root (or any even root) of a negative number. Examples of complex numbers include $-i$, $2 + 3i$, $1 - 2i$.

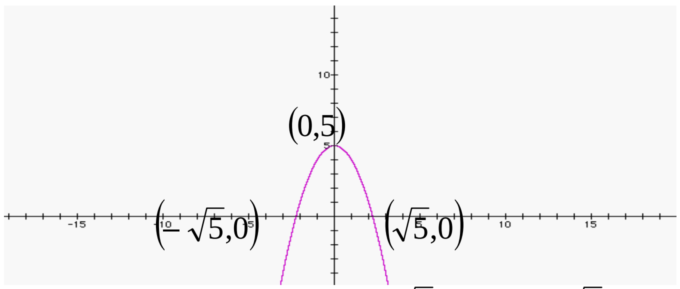
CRUCIAL TERMS

Roots (also called Zeros & Solutions) – the x-values of a polynomial function that give it a y-value equal to zero represent the roots of the polynomial. In symbols, for $P(x)$, if $P(x_1) = 0$, then x_1 is a root. For instance, if $P(2) = 0$, then 2 is a root. Considering $w(x) = x^2 + 6x + 8$, -2 is a root of $w(x)$ because $w(-2) = 0$:

$$w(x) = x^2 + 6x + 8$$
$$w(-2) = (-2)^2 + 6(-2) + 8$$
$$w(-2) = 4 - 12 + 8$$
$$w(-2) = -8 + 8$$
$$w(-2) = 0.$$

Roots can be real or complex. If they are real, roots can be positive or negative, and they can be rational or irrational. Real roots correspond with *x*-intercepts on the graph of the polynomial. Solving the polynomial derives its roots. For example consider $g(x) = 5 - x^2$, solve for the roots of $g(x)$ by setting the function equal to zero:

$$5 - x^2 = 0$$
$$-x^2 = -5$$
$$x^2 = 5$$
$$\sqrt{x^2} = \sqrt{5}$$
$$x = \pm\sqrt{5}$$



The roots of $g(x)$ are $\sqrt{5}$ and $-\sqrt{5}$. Accordingly, $g(x)$ has *x*-intercepts at $(\sqrt{5},0)$ and $(-\sqrt{5},0)$. $\sqrt{5}$ and $-\sqrt{5}$ are irrational roots approximately equal to 2.236 and –2.236. Considering the polynomial $d(t) = -t^2 - 9$, finding the roots of $d(t)$ similarly yields two complex roots:

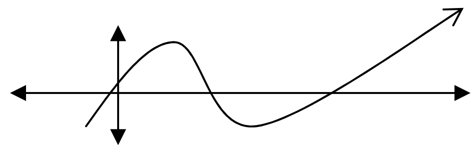
$$-t^2 - 9 = 0$$
$$-t^2 = 9$$
$$t^2 = -9$$
$$\sqrt{t^2} = \sqrt{-9}$$
$$t = \pm 3i$$

The roots then of $d(t)$ are $3i$ and $-3i$. These are complex since they have imaginary parts. Complex roots do *not* appear as *x*-intercepts on the graph of the polynomial. *All roots correspond with **linear factors** (see below).*

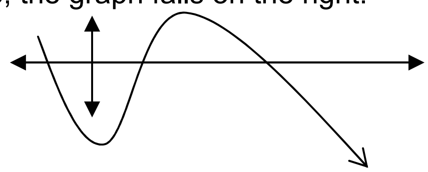
Constant – the constant is the term of a polynomial without a variable part. Consider $K(x) = 7$, $F(x) = x^5 + 4x^4 - 11x^3 - 16x^2 + 58x - 60$, $g(x) = 5 - x^2$, and $j(t) = 2(t + 1)^2(t - 3)^3$. The constant of $K(x)$ is 7. $F(x)$ has a constant equal to –60, and $g(x)$ has a constant equal to 5. The constant for $j(t)$ is not as obvious because $j(t)$ is not written in expanded form. We can determine the constant without expanding the polynomial thusly: $2 \cdot 1^2 \cdot 3^3 = 54$, so 54 is the constant of $j(t)$. The constant corresponds to the *y*-intercept on the graph of the polynomial. Thus, $F(x)$ with a constant of –60 crosses the *y*-axis at (0,-60).

Degree (or Order) – the highest exponent determines the degree, denoted by n, of a polynomial. Consider $K(x) = 7$, $h(x) = 9 + 12x + 4x^2$, $F(x) = x^5 + 4x^4 - 11x^3 - 16x^2 + 58x - 60$, and $v(x) = x^3 - 8$. The degree of $F(x)$ is 5, so $n = 5$. For $h(x)$, the degree is 2, so $n = 2$. Second-degree polynomials are called quadratics. The degree of $v(x)$ is 3. Third-degree polynomials are called cubics. The degree of $K(x)$ is zero. Zero degree polynomials are constant functions. The degree corresponds to the number of roots possessed by a polynomial. Thus, $F(x)$ with a degree of 5 has five roots.

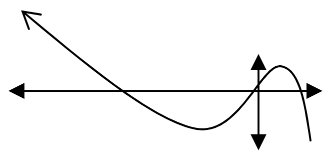
End Behavior – end behavior describes the behavior of the graph to the right (as the x-values approach infinity) and to the left (as the x-values approach negative infinity). End behavior is described as either rising or falling. For right-end behavior, if the function increases as the x-values increase, the graph rises on the right:



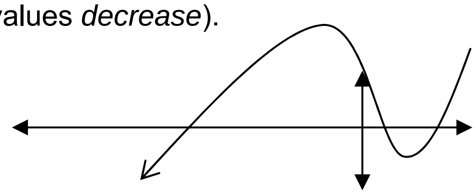
If the function decreases as x-values increase, the graph falls on the right:



For left-end behavior, if the function decreases as x-values increase, the graph rises toward the left (in other words, the function *increases* as x-values *decrease*):



If the function increases as x-values increase, the graph falls toward the left (in other words, the function *decreases* as x-values *decrease*).



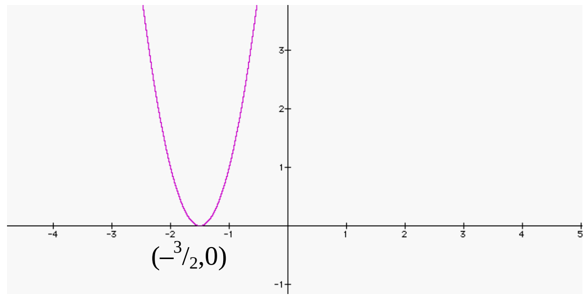
Complex Roots (Non-real Roots) – Complex roots are complex numbers (see above) that give the function a value of zero. Complex roots always occur in conjugate pairs: $a + bi$ and $a - bi$. Complex roots occur in pairs because they are derived from the square root of a negative number (as a result of the quadratic formula), which is preceded by the positive/negative sign ($\pm$).

Leading Coefficient – the leading coefficient is the coefficient to the variable with the highest exponent. Consider $F(x) = x^5 + 4x^4 - 11x^3 - 16x^2 + 58x - 60$, $z(x) = 328 + 78x - 452{,}200x^2$, and $r(t) = 2(5t + 7)^3$. The leading coefficient of $F(x)$ is 1 because 1 is the coefficient to x^5, the highest power of x in F. For $z(x)$ the leading coefficient is –452,200. For $r(t)$ the leading coefficient is equals the product $2 \cdot 5^3$, which is 250.

Linear Factors – the linear factors are the prime factors (allowing for factors with complex numbers) of a polynomial function. Linear factors correspond with the roots of a polynomial (see above). If $w(x) = x^2 + 6x + 8$, the factors of $w(x)$ are $(x + 2)(x + 4)$ because the product of these two binomials equals $x^2 + 6x + 8$. For another example, consider $j(t) = 2(t + 1)^2(t - 3)^3$. Since $j(t)$ is written in factored form, its linear factors, $(t + 1)$ and $(t - 3)$, are easily identified. Each factor corresponds to a root of $j(t)$. The root of $(t + 1)$ is –1. The root of $(t – 3)$ is 3. Since each of these factors is repeated, $(t + 1)$ twice and $(t – 3)$ thrice, the roots possess **multiplicity** (see below). The root –1 has multiplicity two because it comes from a factor repeated twice. The root 3 has multiplicity three because it comes from a factor repeated thrice.

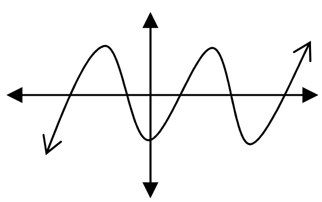
Multiplicity – multiplicity, denoted by m, is the number of repetitions of a given root. For example, the function $h(x) = 9 + 12x + 4x^2$ has a repeated root. Its root comes from the repeated linear factor $(2x + 3)$. Since this linear factor is repeated twice, its corresponding root $-^3/_2$ has multiplicity of two:

$$
\begin{aligned}
&4x^2 + 12x + 9 = 0 \\
&(2x+3)(2x+3) = 0 \\
&2x + 3 = 0 \\
&2x = -3 \\
&x = -\frac{3}{2}\ m2
\end{aligned}
$$



Multiplicity determines a polynomial function's behavior near the x-axis. If the root has even multiplicity, the function will bounce off the x-axis as shown in the graph above. If a root has odd multiplicity, the function will cut through the x-axis.

Turning Points – a turning point is a change in functional behavior from increasing to decreasing or vice versa. If a polynomial has n-distinct roots, it will have at most $n – 1$ turning points. In other words, if a polynomial has 5 distinct roots, it will change increasing/decreasing behavior at most four times:



Turning points comprise a subject for study using the calculus. In College Algebra, the student must simply be cognizant of their existence.

Instruction: *Polynomial Functions, Terminology*

Example 1
Rewriting a Polynomial in Descending Order

Consider $f(x) = 12 - x^2 + 2x^3 - x^4 + 5x^6$.
Rewrite $f(x)$ in $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ form.

$$f(x) = 5x^6 - x^4 + 2x^3 - x^2 + 12$$

Example 2
Identifying the Degree of a Polynomial

Consider $p(x) = 37 + 2x - x^3$. What is the degree of $p(x)$?

For a polynomial of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$, the degree (sometimes called the order) is n, the greatest power on the variable. The degree of $p(x)$ is 3.

Example 3
Identifying the Leading Coefficient of a Polynomial

Consider $p(x) = 37 + 2x - x^3$. What is the leading coefficient of $p(x)$?

For a polynomial of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$, the leading coefficient is a_n, the coefficient to the variable term with the greatest power. The leading coefficient of $p(x)$ is –1.

Example 4
Determining the Number of Roots of a Polynomial

Consider $Y(x) = 5x^2 + 3x$. How many roots does $Y(x)$ have?

A polynomial will have a number of roots (not necessarily distinct or real) equal to its degree.
$Y(x)$ has 2 roots.

Example 5
Determining the Maximum Number of Turning Points of a Polynomial

Consider $P(x) = 7x^5 + 4x^3 - 7x^2 + 2x - 5$. What is the maximum number of turning points that the graph of $P(x)$ will have?

For a *n*-degree polynomial, the maximum number of turning points (points where the function changes in behavior, from increasing to decreasing or from decreasing to increasing) equals $n-1$. $P(x)$ will have a maximum of 4 turning points.

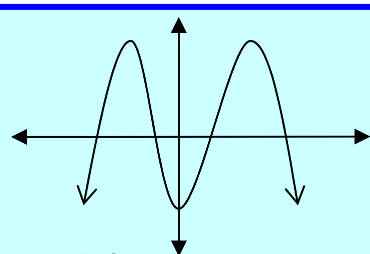
Example 6
Identifying the Constant of a Polynomial

Consider $d(x) = 4x^3 - 5x^2 + 7$. What is the constant of $d(x)$?

For a polynomial of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0$, the constant is a_0, the term without a variable part. The constant of $d(x)$ is 7.

Example 7
Identifying End Behavior

Consider the graph of $T(x)$.

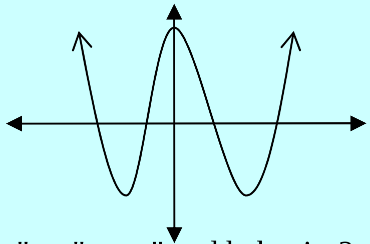


Does the graph "fall" or "rise" on the left?

In terms of left-end behavior, "falling" and "rising" should not be confused with increasing or decreasing behavior. The graph of $T(x)$ "falls" on the left.

Example 8
Identifying End Behavior

Consider the graph of $S(x)$.

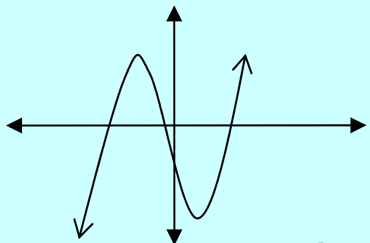


Does the graph exhibit "opposite" or "same" end behavior?

The graph "rises" both on the left and the right ends. The graph exhibits "same" end behavior.

Example 9
Determining the Number of Roots from the Graph

Consider the graph of $F(x)$.

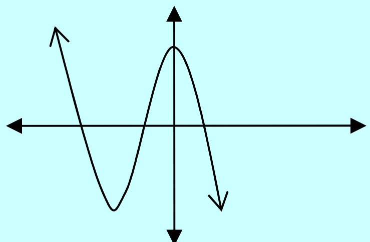


Assume all roots are real and unique. How many roots does $F(x)$ have?

The graph intersects the x-axis three times. If all roots are real and unique, then $F(x)$ must have three roots and be a three-degree polynomial (a cubic).

Example 10
Determining the Constant from the Graph

Consider the graph of $G(x)$.



Is the constant of $G(x)$ greater than or less than zero?

The constant of a polynomial corresponds to its y-intercept. Since the graph intersects the y-axis at a positive value, the function's constant is greater than zero.

Practice Set 2.P

Consider $p(x) = -7x^5 + 2x$ and $q(x) = 3x^2 + 5x - 4$.

1 Which polynomial has the greatest leading coefficient?

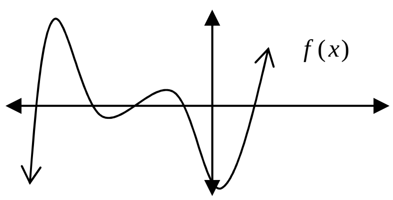
2 Which polynomial has the greatest constant?

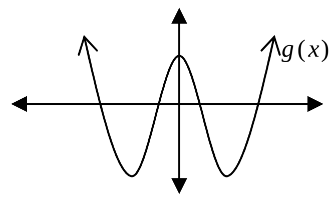
3 Which polynomial has the greatest degree?

4 Which polynomial has the greatest number of linear factors?

5 Which polynomial has the greatest maximum number of turn-arounds?

Consider the graphs.





6 Which polynomial exhibits same end behavior?

7 Which polynomial "falls" on the left?

8 Which polynomial has a positive constant?

9 Which polynomial has the greatest number of real roots assuming all roots are unique?

10 Which polynomial has the least number of positive real roots assuming all roots unique?

1) $3 > -7 \therefore q(x)$

2) $0 > -4 \therefore p(x)$

3) $5 > 2 \therefore p(x)$

4) $5 > 2 \therefore p(x)$

5) $5-1 > 2-1 \therefore p(x)$

6) $g(x)$

7) $f(x)$

8) $g(x)$

9) $f(x)$

10) $f(x)$

Instruction: *Polynomial Functions, Expanded Form and Factored Form*

All polynomials like can be written as a product of linear factors (allowing for factors with complex numbers). Factoring a polynomial to its linear factors sometimes involves synthetic division. The polynomials in this section are readily factored. Consider $f(x) = x^3 - x^2 - 9x + 9$, which can be factored by grouping:

$$f(x) = x^3 - x^2 - 9x + 9$$
$$f(x) = x^2(x - 1) - 9(x - 1)$$
$$f(x) = (x^2 - 9)(x - 1)$$
$$f(x) = (x - 3)(x + 3)(x - 1)$$

Trinomials are sometimes readily factored as well. Consider $p(x) = x^2 - 9x - 190$, which factors to $p(x) = (x + 10)(x - 19)$, and $g(x) = 6x^2 + 29x + 28$, which factors to $g(x) = (2x + 7)(3x + 4)$.

Binomial differences of squares also factor readily. Consider $b(x) = 5x^2 - 45$ factored below:

$$b(x) = 5(x^2 - 9)$$
$$b(x) = 5(x + 3)(x - 3).$$

Of course, a polynomial written as a product of linear factors can be written in expanded form simply by performing the multiplication and finding the product.

Instruction: *Polynomial Functions, Expanded Form and Factored Form*

Example 1
Rewriting a Polynomial in Factored Form

Consider $f(x) = x^2 - \frac{1}{4}$. Rewrite $f(x)$ as a product of linear factors.

Recall that a difference of two squares is a product of two conjugates: $a^2 - b^2 = (a+b)(a-b)$. Note that ½×½ = ¼, and factor the binomial appropriately.

$$f(x) = x^2 - \frac{1}{4}$$
$$f(x) = \left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$$

Example 2
Rewriting a Polynomial in Factored Form

Consider $p(x) = 6x^2 + x - 12$. Rewrite $p(x)$ as a product of linear factors.

Factor the trinomial--that is, find the two binomial factors whose product is the trinomial.

$$p(x) = 6x^2 + x - 12$$
$$p(x) = (2x+3)(3x-4)$$

Example 3
Rewriting a Polynomial in Expanded Form

Consider $P(x) = -2x(3x+1)(5x-6)$. Rewrite $P(x)$ in expanded form.

Multiply the binomials then distribute the $-2x$.

$$p(x) = -2x(3x+1)(5x-6)$$
$$p(x) = -2x\left(15x^2 - 18x + 5x - 6\right)$$
$$p(x) = -2x\left(15x^2 - 13x - 6\right)$$
$$p(x) = -30x^3 + 26x^2 + 12x$$

Practice Set 2.1

Write the following polynomials as a product of their linear factors.

#1 $g(x) = 36 - x^2$

#2 $h(x) = x^2 - 9x + 20$

#3 $j(x) = 3x^2 - 36x + 60$

#4 $K(x) = 6x^2 - 18x$

#5 $m(x) = x^3 + 5x^2 - 4x - 20$

#6 $N(x) = 12x^2 - 32x - 35$

#7 $P(x) = x^3 + 4x^2 - 5x - 20$

#8 $q(x) = 2x^2 - 5x - 3$

#9 $Q(x) = 20x^2 - 7x - 6$

#10 $A(x) = 18x^3 + 63x^2 - 50x - 175$

Write the following polynomial functions in expanded form.

#11 $f(x) = x(7 - x)$

#12 $D(x) = 2(2x + 1)(3x + 1)$

#13 $C(x) = 2x(x + 1)(x - 1)$

#14 $p(x) = (x + 1)(x + 3)(x - 2)$

#1 $g(x) = -1(x + 6)(x - 6)$ or $g(x) = (6 + x)(6 - x)$, #2 $h(x) = (x - 4)(x - 5)$, #3 $j(x) = 3(x - 2)(x - 10)$, #4 $K(x) = 6x(x - 3)$, #5 $m(x) = (x - 2)(x + 2)(x + 5)$, #6 $N(x) = (6x + 5)(2x - 7)$, #7 $P(x) = (x + 4)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)$, #8 $q(x) = (2x + 1)(x - 3)$, #9 $Q(x) = (5x + 2)(4x - 3)$, #10 $A(x) = (3x - 5)(3x + 5)(2x + 7)$, #11 $f(x) = 7x - x^2$, #12 $D(x) = 12x^2 + 10x + 2$, #13 $C(x) = 2x^3 - 2x$, #14 $p(x) = x^3 + 2x^2 - 5x - 6$

Study Exercise 2.1

Problems

Write the following polynomial functions in expanded form.

#1 $P(x) = (4x+1)(x-6)$

#2 $g(x) = x(x+3)(x-3)$

Write the following polynomial functions as a product of their linear factors.

#3 $p(x) = x^2 - 2x - 35$

#4 $f(x) = x^3 - 25x$

Lecture 2.2

College Algebra

Instruction: *Determining Roots from Linear Factors*

Consider $f(x) = (x + 3)(x + 3)(x - 1)$. The roots of $f(x)$ correspond to its linear factors. The roots can be found by setting the polynomial equal to zero and solving for x. For example,

$$(x+3)(x+3)(x-1)=0$$
$$x+3=0, \quad x+3=0, \quad x-1=0$$
$$x=-3, \quad x=-3, \quad x=1$$

If a linear factor is repeated, multiplicity occurs. In the example above, $x = -3$ has a multiplicity of 2.

The Factor Theorem given below refers to the correlation between linear factors and roots shown above.

> **Factor Theorem**: *If* r *is a root of the polynomial* P*(x), then* (x – r) *is a factor of P(x). Conversely, if* (x – r) *is a factor of* P(x)*, then* r *is a root of* P*(x).*

The Factor Theorem correlates the roots of $f(x)$ to its linear factors. Since $x + 3$ is a factor of $f(x)$, then –3 is a root of $f(x)$ by the Factor Theorem. Similarly, since $x - 1$ is a factor of $f(x)$, then 1 is a root of $f(x)$ by the Factor Theorem.

Instruction: ***The Factor Theorem***

Example 1
Finding Roots of Linear Factors

Given $f(x) = -3(22x+1)(4x-5)$, what are the roots of $f(x)$?

Set each linear factor equal to zero and solve. Ignore constant factors.

$$f(x) = -3(22x+1)(4x-5)$$

$$22x+1=0 \qquad 4x-5=0$$
$$22x=-1 \qquad 4x=5$$
$$\frac{22x}{22}=\frac{-1}{22} \qquad \frac{4x}{4}=\frac{5}{4}$$
$$x=-\frac{1}{22} \qquad x=\frac{5}{4}$$

$$roots: -\frac{1}{22}, \frac{5}{4}$$

Example 2
Finding Roots of Linear Factors

Given $p(x) = 5x(2x+3)(x+7)$, what are the roots of $p(x)$?

Set each linear factor equal to zero and solve. Ignore constant factors.

$$p(x) = 5x(2x+3)(x+7)$$

$$x=0 \qquad 2x+3=0 \qquad x+7=0$$
$$2x=-3 \qquad x=-7$$
$$x=-\frac{3}{2}$$

$$roots: 0, -\frac{3}{2}, -7$$

Example 3
Using the Factor Theorem

Recall the Factor Theorem, which states, "*If* r *is a root of the polynomial* P*(x), then* (x – r) *is a factor of P(x). Conversely, if* (x – r) *is a factor of* P(x)*, then* r *is a root of* P(x)." Use the factor theorem to show that a polynomial with the factor $4x+3$ has $-3/4$ as a root.

The factor theorem makes a statement about factors of the form $x-r$. Factor $4x+3$ so that it fits the form of the factors in the factor theorem.

$$4x+3=4\left(x+\frac{3}{4}\right)$$

Since $\left(x+\frac{3}{4}\right)$ is a factor of the polynomial, $-3/4$ must be a root of the polynomial by the factor theorem.

Example 4
Using the Factor Theorem

Recall the Factor Theorem, which states, "*If* r *is a root of the polynomial* P*(x), then* (x – r) *is a factor of P(x). Conversely, if* (x – r) *is a factor of* P(x)*, then* r *is a root of* P(x)." Use the factor theorem to write the expanded form of a polynomial with the roots 2, 5, and –1.

Since 2, 5, and –1 are the factors of the polynomial, then $x-2$, $x-5$, and $x+1$ must be the factors of the polynomial. Multiply the factors to expand the polynomial.

$$P(x)=(x-2)(x-5)(x+1)$$
$$P(x)=(x^2-5x-2x+10)(x+1)$$
$$P(x)=(x^2-7x+10)(x+1)$$
$$P(x)=x^3-7x^2+10x+x^2-7x+10$$
$$P(x)=x^3-6x^2+3x+10$$

Practice Set 2.2A

Identify the roots of each polynomial function.

#1 $F(x) = x(7 - x)$

#2 $D(x) = 2(2x + 1)(3x + 1)$

#3 $C(x) = 2x(x + 1)(x - 1)$

#4 $P(x) = (x + 1)(x + 3)(x - 2)$

#5 $Q(x) = 9(5x + 1)(7x + 3)(5x - 2)$

#6 $R(x) = x(x + 3)(x + 3)(x - 3)$

#7 $S(x) = 2x(4x + 3)(x + 5)(x - 17)$

#8 $G(x) = x - \pi$

#1 $x = 0$ and $x = 7$, #2 $x = -\frac{1}{2}$ and $x = -\frac{1}{3}$, #3 $x = 0$, $x = -1$ and $x = 1$, #4 $x = -1$, $x = -3$ and $x = 2$, #5 $x = -1/5$, $x = -3/7$ and $x = 2/5$, #6 $x = 0$, $x = -3$ and $x = 3$, #7 $x = 0$, $x = -\frac{3}{4}$, $x = -5$ and $x = 17$, #8 $x = \pi$

Practice Set 2.2B

Given the roots of a polynomial, write the polynomial in expanded form. Unless otherwise stated, assume the leading coefficient is one.

#1 *F(x)* has the following roots: $x = 1$.

#2 *P(x)* has the following roots: $x = 7, x = -1$.

#3 *f(x)* has the following roots: $x = 5, x = -5, x = 2$

#4 *p(x)* has the following roots: $x = 1, x = -1, x = \sqrt{3}, x = -\sqrt{3}$.

#5 *g(x)* has the following roots: $x = 4, x = -6, x = 1+\sqrt{2}, x = 1-\sqrt{2}$.

#6 *h(x)* has the following roots: $x = 1, x = 3+i, x = 3-i$.

#7 *v(x)* has the following roots: $x = 1+i\sqrt{2}, x = 1-i\sqrt{2}, x = 0$.

#8 *q(x)* has the following roots: $x = -1, x = 2+\sqrt{5}, x = 2-\sqrt{5}$.

#9 *S(x)* has a leading coefficient of 3 and the following roots: $x = i, x = -i, x = 0, x = \frac{1}{3}$.

#10 *k(x)* has a leading coefficient of 1 and the following roots: $x = i, x = -i, x = 0, x = \frac{1}{3}$.

#11 In its expanded form, *G(x)* has a greatest common factor of 5 and a leading coefficient of 10 and the following roots: $x = 3, x = -2, x = 5, x = \frac{1}{2}$.

#12 *M(x)* has the following roots: $x = 1, x = -3$ (multiplicity 3).

#1 $F(x) = x - 1$

#2 $P(x) = x^2 - 6x - 7$

#3 $f(x) = x^3 - 2x^2 - 25x + 50$

#4 $p(x) = x^4 - 4x^2 + 3$

#5 $g(x) = x^4 - 29x^2 + 46x + 24$

#6 $h(x) = x^3 - 7x^2 + 16x - 10$

#7 $v(x) = x^3 - 2x^2 + 3x$

#8 $q(x) = x^3 - 3x^2 - 5x - 1$

#9 $S(x) = 3x^4 - x^3 + 3x^2 - x$

#10 $k(x) = x^4 - \frac{1}{3}x^3 + x^2 - \frac{1}{3}x$

#11 $G(x) = 10x^4 - 65x^3 + 20x^2 + 305x - 150$

#12 $M(x) = x^4 + 8x^3 + 18x^2 - 27$

Study Exercise 2.2

Problems

Identify the roots of each polynomial function.

#1 $f(x) = x(x+2)$

#2 $p(x) = (x-4)(x-2)(5x+3)(x+5)$