Simplifying Radicals

These notes are to help students simplify radical expressions like the following:

$$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$$

The method of simplifying square root expressions requires the student to factor the radicand (the number under the radical) so that one factor is a perfect square. In the above example, the prime factorization of 12 equals 2·2·3, which can be written as 4·3. Since 4 is a perfect square, its root can be evaluated (taken out of the radical).

EXAMPLE: Simplify $\sqrt{45}$

$\sqrt{45}$ *factor the radicand* $(45 = 3 \cdot 3 \cdot 5)$

$\sqrt{3 \cdot 3 \cdot 5}$ *notice the radicand has a perfect squarefactor* 9

$\sqrt{9 \cdot 5}$ *rewrite as below*

$\sqrt{9} \cdot \sqrt{5}$ *evaluate the root of nine*

$3\sqrt{5}$

Always look for the **largest** perfect square:

EXAMPLE: Simplify $\sqrt{252}$

$\sqrt{252}$ *factor the radicand* $(252 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7)$

notice the radicand has multiple perfect squares in its prime factorization 4 *or* 9 *or* 36

Choose the largest: $\sqrt{36 \cdot 7}$ *instead of* $\sqrt{4 \cdot 63}$ *or* $\sqrt{9 \cdot 28}$.

Then simplify: $\sqrt{36 \cdot 7} = \sqrt{36} \cdot \sqrt{7} = 6\sqrt{7}$.

Sometimes a radical expression will not simplify. For example, the square root of 210, $\left(\sqrt{210}\right)$, does not simplify because 210 does not have any perfect squares in its prime factorization: 210 = 2·3·5·7.

You do not always factor to perfect squares. If the radical indicates cube roots, then we must factor the radicand so that one factor is the largest perfect cube:

EXAMPLE: Simplify $\sqrt[3]{375}$

$\sqrt[3]{375}$ *factor the radicand* $(375 = 3 \cdot 5 \cdot 5 \cdot 5)$

$\sqrt[3]{3 \cdot 5 \cdot 5 \cdot 5}$ *notice the perfect cube* (5^3)

$\sqrt[3]{3 \cdot 125}$ *rewrite as below*

$\sqrt[3]{125} \cdot \sqrt[3]{3}$ *evaluate the root*

$5\sqrt[3]{3}$