

Intermediate Algebra, MATH 0303

Practice Test over

Operations with Rational Expressions & Solving Equations with Rational Expressions

#1) Reduce: $\frac{2x^2 + 7x + 3}{x + 3}$

#2) Reduce: $\frac{6xz + 15x - 14z - 35}{6x^2 + x - 35}$

#3) Divide: $\frac{3x^2 + 7x + 4}{x + 2}$

#4) Divide: $\frac{a^3 - 8}{a - 2}$

#5) Combine: $\frac{2}{x + 3} + \frac{3}{x^2 + 7x + 12} + \frac{1}{x + 4}$

#6) Combine: $\frac{5}{3t} - \frac{4}{t^2}$

#7) Perform the indicated operations: $\frac{y^2 + 2y - 24}{y^2 + 3y + 2} \cdot \frac{y^2 - y - 6}{y^2 + y - 20} \div \frac{y^2 + 3y - 18}{y^2 + 6y + 5}$

#8) Perform the indicated operations: $\frac{2 + y}{y - 4} \cdot \frac{y^2 - 16}{8 + y^3}$

#9) Perform the indicated operations: $\frac{xy + 7x - 3y - 21}{x^2 - 2x - 3} \cdot \frac{x^2 + 4x + 3}{xy + y + 7x + 7}$

#10) Simplify the complex fraction: $\frac{4 - \frac{1}{x^2}}{4 + \frac{4}{x} + \frac{1}{x^2}}$

#11) Simplify the complex fraction:

$$x - \frac{1}{x - \frac{1}{2}}$$

#12) Simplify the complex fraction:

$$\frac{\frac{1}{x+a} + \frac{1}{x-a}}{\frac{1}{x+a} - \frac{1}{x-a}}$$

#13) Solve the equation:

$$\frac{5}{2x} = \frac{2}{x} - \frac{1}{12}$$

#14) Solve the equation:

$$\frac{1}{y+2} - \frac{2}{y-3} = \frac{-2y}{y^2 - y - 6}$$

#15) Solve the equation for y :

$$x = \frac{y-2}{y-3}$$

#16) Solve the equation for P :

$$\frac{1}{P} = \frac{1}{Q} + \frac{1}{R} + \frac{1}{S}$$

SOLUTIONS

#1) Reduce: $\frac{2x^2 + 7x + 3}{x + 3} = \frac{(2x+1)(x+3)}{x+3} = \boxed{2x+1}$

#2) Reduce: $\frac{6xz + 15x - 14z - 35}{6x^2 + x - 35}$
 $\frac{3x(2z+5) - 7(2z+5)}{(2x+5)(3x-7)}$
 $\frac{(3x-7)(2z+5)}{(2x+5)(3x-7)}$

$$\boxed{\frac{2z+5}{2x+5}}$$

$$x+2 \overline{) 3x^2 + 7x + 4}$$

#3) Divide: $\frac{3x^2 + 7x + 4}{x + 2} \rightarrow$

$$\begin{array}{r} 3x^2 + 6x \\ \underline{ + x + 4} \\ x + 2 \\ \underline{ x + 2} \\ 2 \end{array}$$

$$\boxed{3x+1 + \frac{2}{x+2}}$$

$$a-2 \overline{) a^3 + 0a^2 + 0a - 8}$$

$$\underline{a^3 - 2a^2}$$

$$2a^2 + 0a$$

$$\underline{2a^2 - 4a}$$

$$4a - 8$$

$$\underline{4a - 8}$$

$$0$$

#4) Divide: $\frac{a^3 - 8}{a - 2} \Rightarrow$

$$\boxed{a^2 + 2a + 4}$$

$$\frac{2}{x+3} + \frac{3}{x^2 + 7x + 12} + \frac{1}{x+4}$$

$$\frac{2}{x+3} + \frac{3}{(x+3)(x+4)} + \frac{1}{x+4}$$

#5) Combine:

$$\frac{2(x+4)}{(x+3)(x+4)} + \frac{3}{(x+3)(x+4)} + \frac{1(x+3)}{(x+3)(x+4)}$$

$$\frac{2x+8+3+x+3}{(x+3)(x+4)}$$

$\frac{3x+14}{(x+3)(x+4)}$

#6) Combine:

$$\frac{5}{3t} - \frac{4}{t^2}$$

$$\frac{t}{t} \cdot \frac{5}{3t} - \frac{4}{t^2} \cdot \frac{3}{3}$$

$$\frac{5t}{3t^2} - \frac{12}{3t^2}$$

$\frac{5t-12}{3t^2}$

#7) Perform the indicated operations:

$$\frac{y^2 + 2y - 24}{y^2 + 3y + 2} \cdot \frac{y^2 - y - 6}{y^2 + y - 20} \div \frac{y^2 + 3y - 18}{y^2 + 6y + 5}$$

$$\frac{(y+6)(y-4)}{(y+1)(y+2)} \cdot \frac{(y+2)(y-3)}{(y+5)(y-4)} \cdot \frac{y^2 + 6y + 5}{y^2 + 3y - 18}$$

$$\frac{(y+6)(y-3)}{(y+1)(y+5)} \cdot \frac{(y+1)(y+5)}{(y-3)(y+6)}$$

#8) Perform the indicated operations:

$$\frac{2+y}{y-4} \cdot \frac{y^2-16}{8+y^3}$$
$$\frac{2+y}{y-4} \cdot \frac{(y+4)(y-4)}{(2+y)(4-2y+y^2)}$$
$$\frac{(2+y)(y+4)(y-4)}{(y-4)(2+y)(4-2y+y^2)}$$

$$\boxed{\frac{y+4}{4-2y+y^2}}$$

#9) Perform the indicated operations:

$$\frac{xy+7x-3y-21}{x^2-2x-3} \cdot \frac{x^2+4x+3}{xy+y+7x+7}$$
$$\frac{x(y+7)-3(y+7)}{(x+1)(x-3)} \cdot \frac{(x+1)(x+3)}{y(x+1)+7(x+1)}$$
$$\frac{(x-3)(y+7)}{(x+1)(x-3)} \cdot \frac{(x+1)(x+3)}{(y+7)(x+1)}$$

$$\boxed{\frac{x+3}{x+1}}$$

#10) Simplify the complex fraction:

$$\frac{4-\frac{1}{x^2}}{4+\frac{4}{x}+\frac{1}{x^2}} \cdot \frac{\frac{x^2}{1}}{\frac{x^2}{1}}$$
$$\frac{4x^2-1}{4x^2+4x+1}$$
$$\frac{(2x+1)(2x-1)}{(2x+1)(2x+1)}$$

To simplify, multiply the complex fraction by one expressed as the LCD/LCD.

$$\boxed{\frac{2x-1}{2x+1}}$$

#11) Simplify the complex fraction:

$$x - \frac{1}{x - \frac{1}{2}} \quad \leftarrow \begin{array}{|l} \text{Add the} \\ \text{denominator.} \end{array}$$

$$x - \frac{1}{\frac{2x}{2} - \frac{1}{2}}$$

$$x - \frac{1}{\frac{2x-1}{2}}$$

$$x - \frac{2}{2x-1}$$

Remember that 1 divided by a number is equal to 1 times the reciprocal.

$$\frac{2x-1}{2x-1} \cdot \frac{x}{1} - \frac{2}{2x-1}$$

$$\frac{2x^2 - x}{2x-1} - \frac{2}{2x-1}$$

$$\boxed{\frac{2x^2 - x - 2}{2x-1}}$$

To simplify, multiply the complex fraction by one expressed as the LCD/LCD.
Watch out for the subtraction sign, which must be distributed across polynomials.

$$\frac{1}{x+a} + \frac{1}{x-a} \cdot \frac{(x+a)(x-a)}{(x+a)(x-a)} \cdot \frac{1}{(x+a)(x-a)}$$

$$\frac{1}{x+a} - \frac{1}{x-a} \cdot \frac{1}{1}$$

$$\frac{x-a+x+a}{x-a-(x+a)}$$

#12) Simplify the complex fraction:

$$\frac{x+x-a+a}{x-a-x-a}$$

$$\frac{2x}{x-x-a-a}$$

$$\frac{2x}{-2a}$$

$$\boxed{-\frac{x}{a}}$$

To solve equations with rational expressions, multiply the equation by the LCD.

$$\frac{5}{2x} = \frac{2}{x} - \frac{1}{12}$$

$$12x \left(\frac{5}{2x} = \frac{2}{x} - \frac{1}{12} \right)$$

#13) Solve the equation:

$$30 = 24 - x$$

$$6 = -x$$

$$-6 = x$$

$$x = -6$$

#14) Solve the equation:

$$\frac{1}{y+2} - \frac{2}{y-3} = \frac{-2y}{y^2 - y - 6}$$

$$\left(\frac{1}{y+2} - \frac{2}{y-3} = \frac{-2y}{(y+2)(y-3)} \right) \cdot (y+2)(y-3)$$

$$y-3 - 2(y+2) = -2y$$

$$y-3 - 2y - 4 = -2y$$

$$y - 2y - 3 - 4 = -2y$$

$$-y - 7 = -2y$$

$$-7 = -2y + y$$

$$-7 = -y$$

$$7 = y$$

$$y = 7$$

#15) Solve the equation for y:

After multiplying the equation by the LCD, put all the ys on one side of the equation, factor out a y and divide.

$$x = \frac{y-2}{y-3}$$

$$\left(x = \frac{y-2}{y-3} \right) \cdot \frac{y-3}{1}$$

$$x(y-3) = y-2$$

$$xy - 3x = y-2$$

$$xy - y = -2 + 3x$$

$$y(x-1) = -2 + 3x$$

$$y = \frac{-2 + 3x}{x-1}$$

#16) Solve the equation for P :

$$\frac{1}{P} = \frac{1}{Q} + \frac{1}{R} + \frac{1}{S}$$

$$\left(\frac{1}{P} = \frac{1}{Q} + \frac{1}{R} + \frac{1}{S} \right) \cdot \frac{PQRS}{1}$$

$$QRS = PRS + PQS + PQR$$

$$QRS = P(RS + QS + QR)$$

$$\frac{QRS}{RS + QS + QR} = P$$

$$P = \frac{QRS}{RS + QS + QR}$$