Intermediate Algebra (Math 0303)
Practice Test #2

#1 Simplify and write exponents as positive: $(2xy^2z^3)^2(3x^3y^2z)^{-2}$

#2 Simplify and write exponents as positive: $(^1/_2)^{-5} + (^4/_5)^{-2}$

#3 Simplify and write exponents as positive: $\dfrac{(abc^7)^{-1}}{(a^{-1}b^6c^0)^2}$

#4 Simplify and write in scientific notation: $\left[3.2\times10^4\right]\left[1.1\times10^{-6}\right]$

#5 Factor completely: $1 - r^2$

#6 Factor completely: $128 - 2w^6$

#7 Factor completely: $^1/_4x^2 + 3x + 9$

#8 Factor completely: $xyzw + 9xy + zw + 9$

#9 Factor completely: $x^2 - 10x + 25 - y^2$

#10 Factor completely: $5x^2 + 7x - 12$

#11 Solve the equation: $200x^4 = 5000x^2$

#12 Solve the equation: $x^2 = \frac{1}{2}(2x + 24)$

#13 The longest side of a right triangle is two units shorter than double the shortest side. The third side is 8 units long. Find the length of the shortest side.

Practice Test #2
SOLUTIONS

#1 Simplify and write exponents as positive: $(2xy^2z^3)^2(3x^3y^2z)^{-2}$

$$(2xy^2z^3)^2 \cdot \frac{1}{(3x^3y^2z)^2}$$

Apply the property: $a^{-r} = \frac{1}{a^r}$

$$\frac{(2xy^2z^3)^2}{(3x^3y^2z)^2}$$

Apply the properties: $(ab)^r = a^r b^r$

$(a^r)^s = a^{r(s)}$

$$\frac{2^2x^2y^{2(2)}z^{3(2)}}{3^2x^{3(2)}y^{2(2)}z^2}$$

$$\frac{4x^2y^4z^6}{9x^6y^4z^2}$$

$$\frac{4x^{2-6}y^{4-4}z^{6-2}}{9}$$

Apply the property: $\frac{a^r}{a^s} = a^{r-s}$

$$\frac{4x^{-4}y^0z^4}{9}$$

Apply the properties: $a^0 = 1$

$a^{-r} = \frac{1}{a^r}$

$$\boxed{\frac{4z^4}{9x^4}}$$

#2 Simplify and write exponents as positive: $(^1/_2)^{-5} + (^4/_5)^{-2}$

Apply the property: $\left(\frac{a}{b}\right)^{-r} = \left(\frac{b}{a}\right)^4$

$$(2)^5 + \left(\frac{5}{4}\right)^2$$

$$32 + \frac{25}{16}$$

$$32 + 1\frac{9}{16}$$

$$\boxed{33\frac{9}{16}}$$

#3 Simplify and write exponents as positive: $\dfrac{(abc^7)^{-1}}{(a^{-1}b^6c^0)^2}$

$$\frac{a^{-1}b^{-1}c^{7(-1)}}{a^{-1(2)}b^{6(2)}c^{0(2)}}$$

Apply properties: $(ab)^r = a^r b^r$
$(a^r)^s = a^{r\cdot(s)}$

$$\frac{a^{-1}b^{-1}c^{-7}}{a^{-2}b^{12}c^{0}}$$

Apply property: $\dfrac{a^r}{a^s} = a^{r-s}$

$$a^{-1-(-2)}b^{-1-(12)}c^{-7-0}$$

$$a^1b^{-13}c^{-7}$$

$$a \cdot \frac{1}{b^{13}} \cdot \frac{1}{c^7}$$

Apply property: $a^{-r} = \dfrac{1}{a^r}$

$$\boxed{\frac{a}{b^{13}c^7}}$$

#4 Simplify and write in scientific notation: $\left[3.2\times10^4\right]\left[1.1\times10^{-6}\right]$

Apply the associative and commutative properties of multiplication.

$$3.2\times1.1\times10^4\times10^{-6}$$

$$3.51\times10^{4+(-6)}$$

Apply property: $a^r \cdot a^s = a^{r+s}$

$$\boxed{3.51\times10^{-2}}$$

#5 Factor completely: $1 - r^2$

Apply *difference of squares* rule:
$(a^2 - b^2) = (a - b)(a + b)$.

$$\boxed{(1 - r)(1 + r)}$$

#6 Factor completely: $128 - 2w^6$

> This binomial can be factored two different ways. For both factorizations, start by factoring out the greatest common factor. Then apply the three following rules.
>
> *Difference of squares* rule: $a^2 - b^2 = (a-b)(a+b)$
> *Difference of cubes* rule: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
> Sum of cubes rule: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Using Difference of Squares Rule First

$$2(64 - w^6)$$
$$2(8 - w^3)(8 + w^3)$$
$$2(2 - w)(4 + 2w + w^2)(2 + w)(4 - 2w + w^2)$$

Using Difference of Cubes Rule First

$$2(64 - w^6)$$
$$2(4 - w^2)(4 + 4w^2 + w^4)$$
$$2(2 - w)(2 + w)(4 + 4w^2 + w^4)$$

#7 Factor completely: $\frac{1}{4}x^2 + 3x + 9$

Factor the first term: $(\frac{1}{2}x \quad)(\frac{1}{2}x \quad)$
Factor the last term: $(\frac{1}{2}x \ 3)(\frac{1}{2}x \ 3)$

Determine the signs according to the rule: $(\frac{1}{2}x + 3)(\frac{1}{2}x + 3)$
When the second sign of the trinomial is plus then both binomials get the first sign of the trinomial.

Check to see if the middle products (outer *times* outer & inner *times* inner) add to the middle term of the trinomial: $\frac{3}{2}x + \frac{3}{2}x = 3x$

If the middle products add to the middle term, the factorization is correct. Rewrite in simplest form: $(\frac{1}{2}x + 3)^2$

#8 Factor completely: $xyzw + 9xy + zw + 9$

Factor by grouping into 2 binomials. $(xyzw + 9xy) + (zw + 9)$

Factor the greatest common factor of xy from the first binomial and the greatest common factor of 1 from the second binomial. $xy(zw + 9) + 1(zw + 9)$

Rewrite in factored form: $\boxed{(xy + 1)(zw + 9)}$

#9 Factor completely: $x^2 - 10x + 25 - y^2$

Since the first three terms create a perfect square trinomial and since the last term is minus a perfect square, this polynomial is a difference of two squares. To see this clearly, simply factor the first three terms as a trinomial:

$(x^2 - 10x + 25) - y^2$
$(x - 5)(x - 5) - y^2$
$(x - 5)^2 - y^2$

Apply the *difference of squares* rule: $(a^2 - b^2) = (a + b)(a - b)$ $\boxed{(x - 5 + y)(x - 5 - y)}$

#10 Factor completely: $5x^2 + 7x - 12$

Factor the first term: $(5x \quad)(x \quad)$
Factor the last term: $(5x \ \ 12)(x \ \ 1)$

Determine the signs according to the rule: *When the second sign of the trinomial is minus then the largest middle product takes first sign of the trinomial.* $(5x + 12)(x - 1)$

Check to see if the middle products (outer *times* outer & inner *times* inner) add to the middle term of the trinomial: $-5x + 12x = 7x$

If the middle products add to the middle term, the factorization is correct. $\boxed{(5x + 12)(x - 1)}$

#11 Solve the equation: $200x^4 = 5000x^2$

Set the equation equal to zero by subtracting all the terms from one side of the equation:

$$200x^4 - 5000x^2 = 5000x^2 - 5000x^2$$
$$200x^4 - 5000x^2 = 0$$

Factor completely; start by factoring out the greatest common factor:

$$200x^2(x^2 - 25) = 0$$

Factor the difference of two squares:

$$200x^2(x - 5)(x + 5) = 0$$

Factor completely:

$$200xx(x - 5)(x + 5) = 0$$

Set all the factors with a variable equal to zero:

$x = 0$, $x = 0$, $x - 5 = 0$, $x + 5 = 0$

Solve for the variable in each equation:

$x = 0$ (multiplicity 2) $x = 5$ $x = -5$

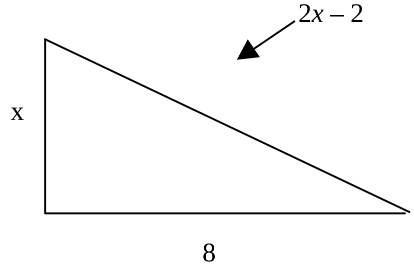
Multiplicity simply refers to how often a given solution occurs. You will learn more about the significance of this term in College Algebra.

#12 Solve the equation: $x^2 = ½(2x + 24)$

Simplify the left by distribution:	$x^2 = x + 12$
Set the equation equal to zero by subtracting all the terms from one side of the equation:	$x^2 - x - 12 = 0$
Factor.	$(x - 4)(x + 3) = 0$
Set each factor with a variable equal to zero:	$x - 4 = 0$, $x + 3 = 0$
Solve:	$x = 4$ $x = -3$

#13 The longest side of a right triangle is two units shorter than double the shortest side. The third side is 8 units long. Find the length of the shortest side.

Draw the figure:



Label the unknown as variable: shortest side equals x
Label knowns: the third side is 8 units long
Label other information: the longest side is $(2x - 2)$

Use Pythagorean theorem to write equation: $x^2 + 8^2 = (2x - 2)^2$
Simplify: $x^2 + 64 = 4x^2 - 8x + 4$
Set equal to zero: $4x^2 - 8x + 4 - x^2 - 64 = 0$
Combine like terms and write in descending order: $3x^2 - 8x - 60 = 0$
Factor: $(3x + 10)(x - 6) = 0$
Set factors equal to zero: $3x + 10 = 0$, $x - 6 = 0$
Solve: $3x = -10$, $x = 6$
$x = -{}^{10}/_{3}$, $x = 6$

Disregard negative since the length must be positive. State the answer.

The shortest side measures 6 units in length.