

## Notes on Solving Equations with Rational Expressions

Step 1: If necessary, factor the denominators to determine the LCD (lowest common denominator). Eliminate rational expressions by multiplying the equation by the LCD. Note, this step is performed properly if, and only if, the rational expressions (fractions) are eliminated.

Step 2: Simplify each side of the equation. Combine like terms.

Step 3: Solve, that is, isolate the variable.

Step 4: Check to be sure that the answer does not make any of the denominators equal to zero.

## Example #1

$$\frac{x}{4} + \frac{2}{3} = \frac{6}{5}$$

The LCD of 4, 3, and 5 is 60. Multiply both sides of the equation by 60.

$$60 \cdot \left( \frac{x}{4} + \frac{2}{3} \right) = \frac{6}{5} \cdot 60$$

$$\frac{60x}{4} + \frac{120}{3} = \frac{360}{5}$$

$$15x + 40 = 72$$

$$15x = 32$$

$$x = \frac{32}{15}$$

## Example #2

Factor  $(x^2 - 1)$   
to  $(x + 1)(x - 1)$ .

$$\frac{x}{x^2 - 1} - \frac{5}{x + 1} = \frac{6}{x - 1}$$

$$\frac{x}{(x + 1)(x - 1)} - \frac{5}{x + 1} = \frac{6}{x - 1}$$

The LCD of  $(x + 1)(x - 1)$ ,  $(x + 1)$ , and  $(x - 1)$  is  $(x + 1)(x - 1)$ . Multiply both sides of the equation by  $(x + 1)(x - 1)$ .

$$(x + 1)(x - 1) \cdot \left( \frac{x}{(x + 1)(x - 1)} - \frac{5}{x + 1} \right) = \frac{6}{x - 1} \cdot (x + 1)(x - 1)$$

$$\frac{x(x + 1)(x - 1)}{(x + 1)(x - 1)} - \frac{5(x + 1)(x - 1)}{x + 1} = \frac{6(x + 1)(x - 1)}{x - 1}$$

$$x - 5(x - 1) = 6(x + 1)$$

$$x - 5x + 5 = 6x + 6$$

$$-4x + 5 = 6x + 6$$

$$5 - 6 = 6x + 4x$$

$$-1 = 10x$$

$$-\frac{1}{10} = x$$

$$-.1 = x$$