Math for Business and Social Sciences II
Practice Test 3

#1 Evaluate $\int \left(1 + x + e^{x}\right) dx$

#2 Evaluate $\int \frac{1}{x^{2}} \left(x^{4} - 2x^{2} + 1\right) dx$

#3 Evaluate $\int \frac{x^{2}}{\sqrt{x^{3} - 1}} dx$

#4 Evaluate $\int \frac{e^{2x}}{1 + e^{2x}} dx$

#5 Find the area under the curve $f(x) = \frac{1}{\sqrt{x}}$ on the interval [1,9].

#6 Evaluate $\int_1^8 4x^{1/3}\,dx$

#7 Evaluate $\int_1^4 \left(\frac{3x^3 - 2x^2 + 4}{x^2}\right)dx$

#8 Evaluate $\int_0^2 xe^{x^2}\,dx$

#9 Find the average value of the function $f(x) = 4 - x^2$ on the interval $[-2,3]$.

#10 Evaluate using integration by parts: $\int 6xe^{3x}\,dx$

#11 Find the area of the region bounded by the graphs of $f(x) = x^2 + 2$ and $g(x) = 1 - x$ and the vertical lines $x = 0$ and $x = 1$.

#12 Find the area of the region bounded by the graphs of $f(x) = 9 - x^2$, $g(x) = 2x + 3$, $x = 1$, and $x = -1$.

SOLUTIONS

#1 Evaluate $\int \left(1+x+e^{x}\right)dx$

$$\int \left(1+x+e^{x}\right)dx$$

$$x+\frac{1}{2}x^{2}+e^{x}+C$$

$$\boxed{\frac{1}{2}x^{2}+x+e^{x}+C}$$

#2 Evaluate $\int \frac{1}{x^{2}}\left(x^{4}-2x^{2}+1\right)dx$

$$\int \frac{1}{x^{2}}\left(x^{4}-2x^{2}+1\right)dx$$

$$\int x^{-2}\left(x^{4}-2x^{2}+1\right)dx$$

$$\int \left(x^{2}-2+x^{-2}\right)dx$$

$$\frac{1}{3}x^{3}-2x-x^{-1}+C$$

$$\boxed{\frac{1}{3}x^{3}-2x-\frac{1}{x}+C}$$

#3 Evaluate $\int \frac{x^{2}}{\sqrt{x^{3}-1}}dx$

$$\int \frac{x^{2}}{\left(x^{3}-1\right)^{\frac{1}{2}}}dx$$

Let $u = x^{3}-1$ so that $du = 3x^{2}dx$. Solve for $x^{2}dx$:
$du = 3x^{2}dx$
$\frac{1}{3}du = x^{2}dx$

$$\frac{1}{3}\int \frac{du}{\left(u\right)^{\frac{1}{2}}}=\frac{1}{3}\int u^{-\frac{1}{2}}du=\frac{2}{3}u^{\frac{1}{2}}+C$$

$$\boxed{\frac{2}{3}\sqrt{x^{3}-1}+C}$$

#4 Evaluate $\int \frac{e^{2x}}{1+e^{2x}}dx$

Let $u = 1 + e^{2x}$ so that $du = 2e^{2x}dx$. Solve for $e^{2x}dx$.
$du = 2e^{2x}dx$
$\frac{1}{2}du = e^{2x}dx$

$$\frac{1}{2}\int \frac{du}{u}=\frac{1}{2}\ln\left|u\right|+C=\frac{1}{2}\ln\left(1+e^{2x}\right)+C$$

$$\boxed{\frac{1}{2}\ln\left(1+e^{2x}\right)+C}$$

#5 Find the area under the curve $f(x) = \frac{1}{\sqrt{x}}$ on the interval [1,9].

$$\int_1^9 x^{-\frac{1}{2}} dx$$

$$2x^{\frac{1}{2}} + C - \left(2x^{\frac{1}{2}} + C \right) \Big|_1^9$$

$$2(9)^{\frac{1}{2}} + C - 2(1)^{\frac{1}{2}} - C$$

$$2 \cdot 3 - 2 \cdot 1$$

$$6 - 2$$

4 sq. units

#6 Evaluate $\int_1^8 4x^{1/3} dx$

$$\int_1^8 4x^{1/3} dx$$

$$\frac{3}{4} \cdot 4x^{4/3} + C \Big|_1^8$$

$$3x^{4/3} + C - [3x^{4/3} + C] \Big|_1^8$$

$$3(8)^{4/3} + C - [3(1)^{4/3} + C]$$

$$3(2)^4 + C - [3 \cdot 1 + C]$$

$$3 \cdot 16 + C - 3 - C$$

$$48 - 3$$

45

#7 Evaluate $\int_1^4 \left(\frac{3x^3 - 2x^2 + 4}{x^2} \right) dx$

$$\int_1^4 \left(\frac{3x^3 - 2x^2 + 4}{x^2} \right) dx$$

$$\int_1^4 x^{-2}(3x^3 - 2x^2 + 4) dx$$

$$\int_1^4 \left(3x - 2 + 4x^{-2} \right)$$

$$\frac{1}{2} \cdot 3x^2 - 2x + -1 \cdot 4x^{-1} + C \Big|_1^4$$

$$\frac{3}{2}x^2 - 2x - \frac{4}{x} + C - \left[\frac{3}{2}x^2 - 2x - \frac{4}{x} + C \right] \Big|_1^4$$

$$\frac{3}{2}(4)^2 - 2(4) - \frac{4}{4} + C - \left[\frac{3}{2}(1)^2 - 2(1) - \frac{4}{1} + C \right]$$

$$\frac{3}{2} \cdot 16 - 8 - 1 + C - \left[\frac{3}{2} - 2 - 4 + C \right]$$

$$24 - 8 - 1 + C - \frac{3}{2} + 2 + 4 - C$$

$$15 - \frac{3}{2} + 6$$

$\frac{39}{2}$

#8 Evaluate $\int_0^2 xe^{x^2}\,dx$

Let $u = x^2$ If $x = 0$, $u = 0$, and if $x = 2$, $u = 4$.
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$\int_0^2 xe^{x^2}\,dx = \int_0^4 e^u\,du = \frac{1}{2}e^u \Big|_0^4 = \frac{1}{2}e^4 - \frac{1}{2}e^0 = \frac{1}{2}e^4 - \frac{1}{2}\cdot 1 = \frac{1}{2}e^4 - \frac{1}{2} = \boxed{\frac{1}{2}\left(e^4 - 1\right)}$$

#9 Find the average value of the function
$f(x) = 4 - x^2$ on the interval $[-2,3]$.

$$AV = \frac{1}{5}\int_{-2}^{3}\left(4 - x^2\right)dx$$

$$AV = \frac{1}{5}\left(4x - \frac{1}{3}x^3\right)\Big|_{-2}^{3}$$

$$AV = \frac{1}{5}\left[\left(4x - \frac{1}{3}x^3\right) - \left(4x - \frac{1}{3}x^3\right)\right]\Big|_{-2}^{3}$$

$$AV = \frac{1}{5}\left[\left(4\cdot 3 - \frac{1}{3}\cdot 3^3\right) - \left(4(-2) - \frac{1}{3}(-2)^3\right)\right]$$

$$AV = \frac{1}{5}\left[\left(12 - \frac{1}{3}\cdot 27\right) - \left(-8 - \frac{1}{3}\cdot -8\right)\right]$$

$$AV = \frac{1}{5}\left[(12 - 9) - (-8 + \frac{8}{3}\right]$$

$$AV = \frac{1}{5}\left[3 - \left(-\frac{16}{3}\right)\right]$$

$$AV = \frac{1}{5}\left(\frac{25}{3}\right)$$

$$\boxed{AV = \frac{5}{3}}$$

#10 Evaluate using integration by parts: $\int 6xe^{3x}\,dx$

Let $u = 6x$ and $dv = e^{3x}\,dx$.
then $du = 6\,dx$ and $v = \frac{1}{3}e^{3x}$.

$$\text{Let } I = \int 6xe^{3x}\,dx$$
$$\text{then } I = 2xe^{3x} - \int 2e^{3x}\,dx$$
$$I = 2xe^{3x} - 2\int e^{3x}\,dx$$
$$I = 2xe^{3x} - 2\cdot\frac{1}{3}e^{3x} + C$$
$$\boxed{I = \frac{2}{3}e^{3x}(3x-1) + C}$$

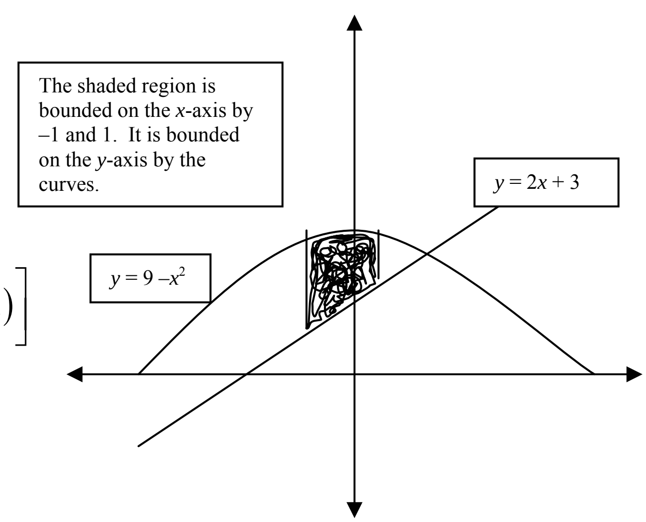
#11 Find the area of the region bounded by the graphs of $f(x) = x^2 + 2$ and $g(x) = 1 - x$ and the vertical lines $x = 0$ and $x = 1$.

$$\int_0^1 \left[\left(x^2+2\right)-\left(1-x\right)\right]dx = \int_0^1 \left(x^2+x+1\right)dx = \frac{1}{3}x^3+\frac{1}{2}x^2+x\Big|_0^1 = \frac{1}{3}+\frac{1}{2}+1-0 = \boxed{\frac{11}{6}}$$

#12 Find the area of the region bounded by the graphs of $f(x) = 9 - x^2$, $g(x) = 2x + 3$, $x = 1$, and $x = -1$.

The region is shown in the figure.

$$A = \int_{-1}^{1}\left[\left(9-x^2\right)-\left(2x+3\right)\right]dx$$
$$A = \int_{-1}^{1}\left[-x^2-2x+6\right]dx$$
$$A = -\frac{1}{3}x^3 - x^2 + 6x\Big|_{-1}^{1}$$
$$A = -\frac{1}{3}\cdot 1^3 - 1^2 + 6\cdot 1 - \left[-\frac{1}{3}(-1)^3 - (-1)^2 + 6(-1)\right]$$
$$A = -\frac{1}{3} - 1 + 6 - \left[\frac{1}{3} - 1 - 6\right]$$
$$A = -\frac{4}{3} + 6 - \frac{1}{3} + 1 + 6$$



$$\boxed{A = \frac{34}{3}}$$