Math 1325
Practice Test #1

#1) Find: $\lim_{x\to 2} \frac{x^2+10}{x-4}$.

#2) Find: $\lim_{x\to 5} \frac{x^2-4x-5}{x^2+2x-35}$

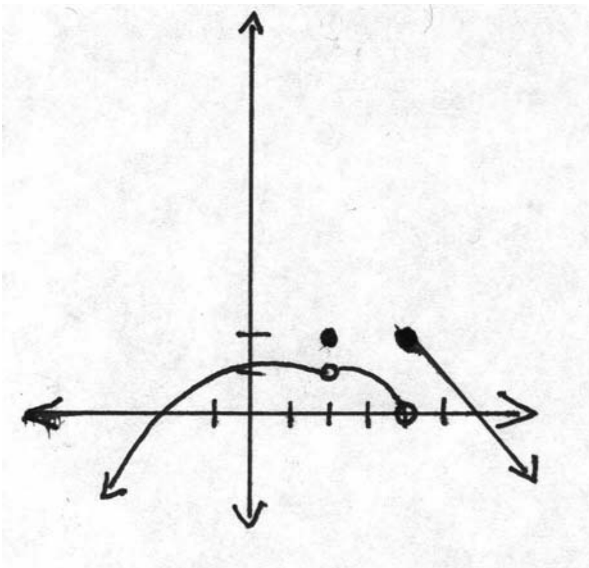
#3) Find: $\lim_{x\to 3} \frac{\sqrt{25-h^2}-4}{3-h}$.

#4) Find: $\lim_{x\to \infty} \frac{4x^2+8}{8x^2+2x-1}$.

#5) Find: $\lim_{x\to \infty} \frac{x+8}{x^2-11}$

#6) Find: $\lim_{x\to-\infty} \dfrac{x^3+x+1}{x^2-3}$.

#7) Consider the function graphed below.



#7 Part A) Find: $\lim_{x\to 2} f(x)$

#7 Part B) Find: $\lim_{x\to 4^-} f(x)$

#7 Part C) Find: $\lim_{x\to 4^+} f(x)$

#7 Part D) Find: $\lim_{x\to 4} f(x)$

#7 Part E) Find: $f(2)$

#8) Name the intervals of continuity for the function: $f(x) = \dfrac{x+1}{x^2 - 3x + 2}$.

#9) Use the definition of a derivative, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$, to find $f'(x)$ for $f(x) = x^2 + x$.

#10) Find $f'(x)$ if $f(x) = 4x^{\frac{5}{4}}$.

#11) Find $f'(x)$ if $f(x) = 8x^4 + 6x^3 - 12x^2 - 20x + 3$.

#12) Find the equation of the line tangent to the graph of $f(x) = 3x^2 - x + 1$ when $x = 1$.

#13) Find the equation of the line tangent to $g(x) = 1 - x^2$ at $x = -1$.

#14) Explain why $r(x) = \dfrac{x+1}{x^2 - 1}$ does not have a tangent line at $x = -1$.

#15) Given $f(x) = 2x \ln x$, find $f'(x)$.

#16) Given $y(x) = e^{2x}$, find $y'(x)$.

#17) Given $g(x) = \dfrac{6x}{x^2 + 2}$, find $g'(x)$.

#18 Given $p(x) = \left(5x^2 + 2\right)^9$, find $p'(x)$.

#19 Given $h(x) = 4x^2\left(6x + 7\right)^6$, find $h'(x)$.

#20 Given $y(x) = 9x^2$, find $y''(x)$.

SOLUTIONS

#1) Find: $\lim_{x\to 2} \frac{x^2+10}{x-4} = \frac{(2)^2+10}{2-4} = \frac{14}{-2} = \boxed{-7}$.

#2) Find: $\lim_{x\to 5} \frac{x^2-4x-5}{x^2+2x-35} = \frac{(x+1)\cancel{(x-5)}}{(x+7)\cancel{(x-5)}} = \frac{x+1}{x+7} = \frac{5+1}{5+7} = \frac{6}{12} = \boxed{\frac{1}{2}}$

$\frac{5^2-4\cdot 5-5}{5^2+2\cdot 5-35} = \frac{25-20-5}{25+10-35} = \frac{0}{0}$ Indeterminant

#3) Find: $\lim_{x\to 3} \frac{\sqrt{25-h^2}-4}{3-h} \cdot \frac{\sqrt{25-h^2}+4}{\sqrt{25-h^2}+4}$

Rationalize the numerator. Use the difference of squares rule: $(a+b)(a-b) = a^2-b^2$.

$$\lim_{x\to 3} \frac{\sqrt{25-h^2}-4}{3-h} \cdot \frac{\sqrt{25-h^2}+4}{\sqrt{25-h^2}+4}$$

$$\lim_{x\to 3} \frac{25-h^2-16}{(3-h)\left(\sqrt{25-h^2}+4\right)}$$

$$\lim_{x\to 3} \frac{25-16-h^2}{(3-h)\left(\sqrt{25-h^2}+4\right)}$$

$$\lim_{x\to 3} \frac{9-h^2}{(3-h)\left(\sqrt{25-h^2}+4\right)}$$

$$\lim_{x\to 3} \frac{(3+h)(3-h)}{(3-h)\left(\sqrt{25-h^2}+4\right)}$$

$$\lim_{x\to 3} \frac{(3+h)}{\sqrt{25-h^2}+4} = \frac{3+3}{\sqrt{25-(3)^2}+4} = \frac{6}{\sqrt{16}+4} = \frac{6}{4+4} = \frac{6}{8} = \boxed{\frac{3}{4}}$$

#4) Find: $\lim_{x\to\infty} \frac{4x^2+8}{8x^2+2x-1}$.

$$\lim_{x\to\infty} \frac{\frac{4x^2}{x^2}+\frac{8}{x^2}}{\frac{8x^2}{x^2}+\frac{2x}{x^2}-\frac{1}{x^2}} = \frac{4+\frac{8}{x^2}}{8+\frac{2}{x}-\frac{1}{x^2}} = \frac{4+0}{8+0+0} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

#5) Find: $\lim_{x\to\infty} \frac{x+8}{x^2-11}$

$$\lim_{x\to\infty} \frac{x+8}{x^2-11} = \frac{\frac{x}{x^2}+\frac{8}{x^2}}{\frac{x^2}{x^2}-\frac{11}{x^2}} = \frac{\frac{1}{x}+\frac{8}{x^2}}{1-\frac{11}{x^2}} = \frac{0+0}{1-0} = \frac{0}{1} = \boxed{0}$$

#6) Find: $\lim_{x\to-\infty} \frac{x^3+x+1}{x^2-3}$.

$$\lim_{x\to-\infty} \frac{x^3+x+1}{x^2-3}$$

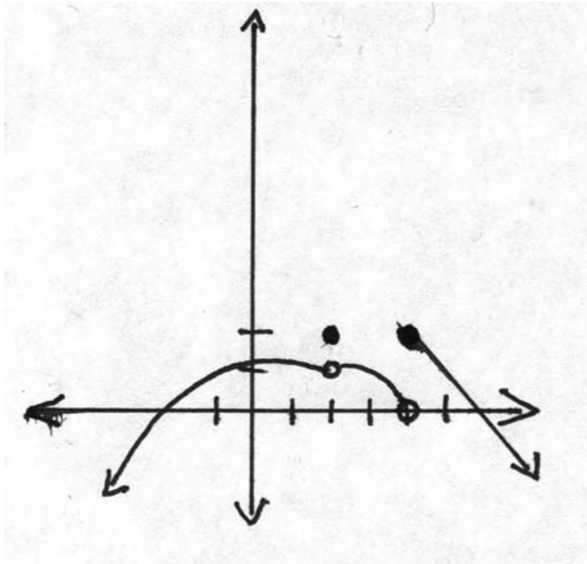
$$\lim_{x\to-\infty} \frac{\frac{x^3}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{3}{x^2}}$$

$$\lim_{x\to-\infty} \frac{x+\frac{1}{x}+\frac{1}{x^2}}{1-\frac{3}{x^2}}$$

$$\lim_{x\to-\infty} \frac{x+0+0}{1-0}$$

$$\lim_{x\to-\infty} x = \boxed{-\infty}$$

#7) Consider the function graphed below.



#7 Part A) Find: $\lim_{x\to 2} f(x) = 1$

#7 Part B) Find: $\lim_{x\to 4^-} f(x) = 0$

#7 Part C) Find: $\lim_{x\to 4^+} f(x) = 2$

#7 Part D) Find: $\lim_{x\to 4} f(x) =$ does not exist

#7 Part E) Find: $f(2) = 2$

#8) Name the intervals of continuity for the function: $f(x)=\dfrac{x+1}{x^2-3x+2}$.

The function will be discontinuous if the denominator equals zero. To find the *x*-values where the function is discontinuous, set the denominator equal to zero and solve for *x*.

$$x^2-3x+2=0$$
$$(x-1)(x-2)=0$$
$$x-1=0 \quad x-2=0$$
$$x=1 \qquad x=2$$

Intervals of continuity: $(-\infty,1)\bigcup(1,2)\bigcup(2,\infty)$

#9) Use the definition of a derivative, $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, to find $f'(x)$ for $f(x)=x^2+x$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$\lim_{h\to 0}\frac{(x+h)^2+x+h-\left[x^2+x\right]}{h}$$
$$\lim_{h\to 0}\frac{(x+h)(x+h)+x+h-x^2-x}{h}$$
$$\lim_{h\to 0}\frac{x^2+xh+xh+h^2+x+h-x^2-x}{h}$$
$$\lim_{h\to 0}\frac{x^2+2xh+h^2+x+h-x^2-x}{h}$$
$$\lim_{h\to 0}\frac{x^2-x^2+2xh+h^2+x-x+h}{h}$$
$$\lim_{h\to 0}\frac{2xh+h^2+h}{h}$$
$$\lim_{h\to 0}\frac{\cancel{h}(2x+h+1)}{\cancel{h}}$$
$$\lim_{h\to 0}2x+h+1$$

$$2x+0+1$$

$f'(x)=2x+1$

#10) Find $f'(x)$ if $f(x) = 4x^{\frac{5}{4}}$.

$$f(x) = 4x^{\frac{5}{4}}$$
$$f'(x) = \frac{5}{4} \cdot 4x^{\frac{5}{4}-\frac{4}{4}}$$
$$f'(x) = 5x^{\frac{1}{4}}$$

#11) Find $f'(x)$ if $f(x) = 8x^4 + 6x^3 - 12x^2 - 20x + 3$.

$$f(x) = 8x^4 + 6x^3 - 12x^2 - 20x + 3$$

$$f'(x) = 4 \cdot 8x^3 + 3 \cdot 6x^2 - 2 \cdot 12x - 20$$
$$f'(x) = 32x^3 + 18x^2 - 24x - 20$$

#12) Find the equation of the line tangent to the graph of $f(x) = 3x^2 - x + 1$ when $x = 1$.

$$f(x) = 3x^2 - x + 1$$
$$f'(x) = 6x - 1$$

Find the derivative, which is a formula for the slope of a tangent line at any given point.

$$f'(1) = 6 \cdot 1 - 1$$
$$f'(1) = 5$$

Use the derivative to find the slope of the tangent line when x equals one.

$$f(1) = 3(1)^2 - 1 + 1$$
$$f(1) = 3$$

Find the point on the function that the tangent line passes through.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 5(x - 1)$$
$$y - 3 = 5x - 5$$
$$y = 5x - 5 + 3$$
$$y = 5x - 2$$

Substitute the slope of the tangent line and the point of tangency into the point-slope formula to find the equation of the tangent line.

#13) Find the equation of the line tangent to $g(x) = 1 - x^2$ at $x = -1$.

$g(x) = 1 - x^2$
$g'(x) = -2x$

Find the derivative, which is a formula for the slope of a tangent line at any given point.

$g'(-1) = -2(-1)$
$g'(-1) = 2$

Use the derivative to find the slope of the tangent line when x equals one.

$g(-1) = 1 - (-1)^2$
$g(-1) = 0$

Find the point on the function that the tangent line passes through.

$y - y_1 = m(x - x_1)$
$y - 0 = 2(x - -1)$
$y = 2(x + 1)$
$y = 2x + 2$

Substitute the slope of the tangent line and the point of tangency into the point-slope formula to find the equation of the tangent line.

#14) Explain why $r(x) = \dfrac{x+1}{x^2 - 1}$ does not have a tangent line at $x = -1$.

The function is undefined when x equals -1; therefore, a line cannot be tangent to the graph at some point where x is -1 since the graph does not exist there.

#15) Given $f(x) = 2x \ln x$, find $f'(x)$.

$$f(x) = 2x \ln x$$
$$f'(x) = 2x \cdot \frac{dy}{dx} \ln x + \ln x \cdot \frac{dy}{dx} 2x$$
$$f'(x) = 2x \cdot \frac{1}{x} + 2 \cdot \ln x$$
$$f'(x) = 2 + 2 \cdot \ln x$$
$$f'(x) = 2 + \ln x^2$$

#16) Given $y(x) = e^{2x}$, find $y'(x)$.

$$y(x) = e^{2x}$$
$$y'(x) = e^{2x} \cdot \frac{dy}{dx} 2x$$
$$y'(x) = 2e^{2x}$$

#17) Given $g(x)=\frac{6x}{x^2+2}$, find $g'(x)$.

$$g(x)=\frac{6x}{x^2+2}$$

$$g'(x)=\frac{\left(x^2+2\right)\cdot\frac{dy}{dx}6x-\left[6x\cdot\frac{dy}{dx}\left(x^2+2\right)\right]}{\left(x^2+2\right)^2}$$

$$g'(x)=\frac{\left(x^2+2\right)\cdot 6-\left[6x\cdot 2x\right]}{\left(x^2+2\right)^2}$$

$$g'(x)=\frac{6x^2+12-\left[12x^2\right]}{\left(x^2+2\right)^2}$$

$$g'(x)=\frac{-6x^2+12}{\left(x^2+2\right)^2}$$

$$g'(x)=\frac{-6\left(x^2-2\right)}{\left(x^2+2\right)^2}$$

#18 Given $p(x)=\left(5x^2+2\right)^9$, find $p'(x)$.

$$p(x)=\left(5x^2+2\right)^9$$

$$p'(x)=9\left(5x^2+2\right)^8\cdot\frac{dy}{dx}\left(5x^2+2\right)$$

$$p'(x)=9\left(5x^2+2\right)^8\cdot\left(10x\right)$$

$$p'(x)=90x\left(5x^2+2\right)^8$$

#19 Given $h(x)=4x^2(6x+7)^6$, find $h'(x)$.

$$h(x)=4x^2(6x+7)^6$$

$$h'(x)=4x^2\cdot\frac{dy}{dx}(6x+7)^6+(6x+7)^6\cdot\frac{dy}{dx}4x^2$$

$$h'(x)=4x^2\cdot 6(6x+7)^5\cdot\frac{dy}{dx}(6x+7)+(6x+7)^6\cdot 8x$$

$$h'(x)=24x^2(6x+7)^5\cdot 6+8x(6x+7)^6$$

$$h'(x)=144x^2(6x+7)^5+8x(6x+7)^6$$

$$h'(x)=8x(6x+7)^5\left[18x+(6x+7)\right]$$

$$h'(x)=8x(6x+7)^5(24x+7)$$

$$h'(x)=8x(24x+7)(6x+7)^5$$

#20 Given $y(x) = 9x^2$, find $y''(x)$.

$$y(x)=9x^2$$

$$y'(x)=18x$$

$$y''(x)=18$$