RATIONAL FUNCTIONS

7 Steps To Graphing a Rational Function

Example with smaller degree in numerator	Example with degree of numerator equal to degree of denominator	Example with larger degree in numerator
$f(x)=\frac{x+2}{x^2-4}$	$g(x)=\frac{2x}{x-1}$	$h(x)=\frac{x^2-4x-5}{x-3}$

1. The domain of rational functions includes all real numbers except where the denominator equals zero.

$x^2-4=0$ $(x+2)(x-2)=0$ $x+2=0$ $x-2=0$ $x=-2$ $x=2$	$x-1=0$ $x=1$	$x-3=0$ $x=3$
domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$	domain: $(-\infty,1)\cup(1,\infty)$	domain: $(-\infty,3)\cup(3,\infty)$

2. The vertical asymptotes of rational functions exist as vertical lines with the equation(s) x=a where "a" represents the values for "x" where the denominator equals zero AFTER common factors of the numerator and denominator have been cancelled.

$f(x)=\frac{x+2}{(x+2)(x-2)}$ $f(x)=\frac{1}{x-2}\ x\_-2$	$g(x)=\frac{2x}{x-1}$	$h(x)=\frac{(x+1)(x-5)}{x-3}$
VA: x=2	VA: x=1	VA x=3

3. The horizontal asymptotes of rational functions appear with equation(s) y=b according to the following rules:

If the degree of the numerator is smaller than the degree of the denominator, then y=0. $f(x)=\frac{x+2}{x^2-4}$	If the degree of the numerator is equal to the degree of the denominator, then $y=\frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$ $g(x)=\frac{2x}{x-1}$	If the degree of the numerator is larger than the degree of the denominator, then there is no horizontal asymptote. $h(x)=\frac{x^2-4x-5}{x-3}$
HA: y=0	HA: y=2	HA: NONE

4. Slant asymptotes appear only in rational functions where the degree of the numerator is exactly one larger than the degree of the denominator. Slant asymptotes appear with equations y=the quotient without the remainder.

$$x-3\overline{)x^2-4x-5}$$ quotient: x - 1

x - 1

$-x^2+3x$

-x-5

+x+-3

-8

NONE	NONE	SA: y=x-1

5. Y-intercepts appear (if there is one) where x=0.

$$f(0)=\frac{0+2}{0^2-4}=\frac{2}{-4}=-\frac{1}{2}$$

$$g(0)=\frac{2(0)}{0-1}=\frac{0}{-1}=0$$

$$h(0)=\frac{0^2-4(0)-5}{0-3}=\frac{-5}{-3}=\frac{5}{3}$$

y-intercept: (0,-1/2)	y-intercept: (0,0)	y-intercept: (0,5/3)

6. X-intercepts appear (if there is one) where the function equals zero.

$$f(x)=\frac{x+2}{x^2-4}$$

$$g(x)=\frac{2x}{x-1}$$

$$h(x)=\frac{x^2-4x-5}{x-3}$$

$$(x^2-4)\bullet(\frac{x+2}{x^2-4})=(0)\bullet(x^2-4)$$

$$(x-1)\bullet(\frac{2x}{x-1})=(0)\bullet(x-1)$$

$$(x-3)\bullet(\frac{x^2-4x-5}{x-3})=(0)\bullet(x-3)$$

x+2=0
x=-2
But, remember that -2 is not part of the domain, so there is no x-intercepts.

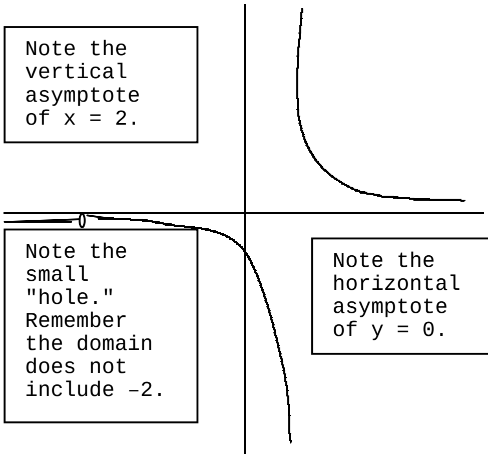
2x=0
x=0

x^2-4x-5=0
(x+1)(x-5)=0
x+1=0 x-5=0
x=-1 x=5

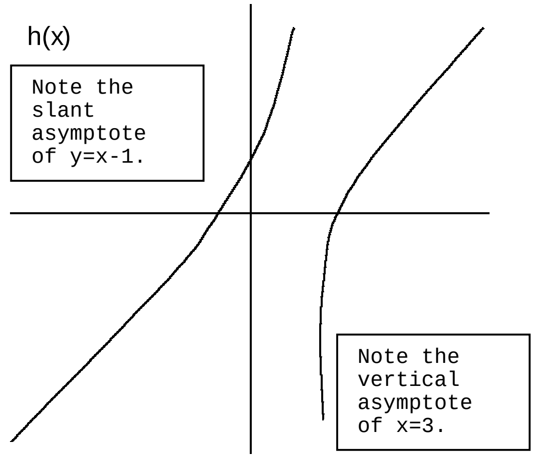
x-intercepts: NONE	x-intercepts: (0,0)	x-intercepts: (-1,0) and (5,0)

7. A T-table will help graph the function.

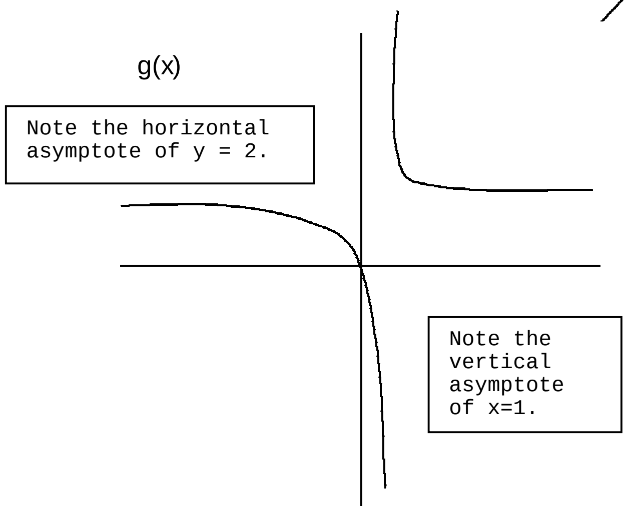
f(x)



h(x)



g(x)



f(x)=

x	y
-4	-1/6
-3	-1/5
-5/2	-2/9
-2	undefined
-1	-1/3
-1/2	-2/5
0	-1/2
1/2	-2/3
1	-1
3/2	-2
2	undefined
5/2	2
3	1
7/2	2/3
4	1/2

g(x)=

x	y
-2	4/3
-3/2	6/5
-1	1
-1/2	2/3
0	0
1/2	-2
1	undefined
3/2	6
2	4
5/2	10/3

h(x)=

x	y
-5	-5
-4	-4.43
-3	-8/3
-2	-7/5
-1	0
0	5/3
1/2	27/10
1	4
3/2	5.83
2	9
3	undefined
4	-5
5	0
6	7/3