

Functions and Functional Notation

$y = x^2 + 7x + 12$ is a function because one variable (y) is dependent upon another variable (x). A function can be written in functional notation as: $f(x) = x^2 + 7x + 12$, meaning that the dependent variable is a function of the x -variable. The “ $f(x)$ ” still stands for the dependent variable (usually “ y ”). In functional notation, the number inside the parenthesis next to the letter (usually “ f ”) represents the value to be substituted in for the independent variable (usually “ x ”). So, $f(4)$ means substitute 4 in for x :

$$\begin{aligned} f(x) &= x^2 + 7x + 12 \\ f(4) &= 4^2 + 7(4) + 12 \\ f(4) &= 16 + 28 + 12 \\ f(4) &= 56 \end{aligned} \qquad \text{the function goes through the point } (4,56).$$

Functional notation can create complicated expressions. For example, $f(a + h)$ means substitute “ $a + h$ ” for x :

$$\begin{aligned} f(a + h) &= (a + h)^2 + 7(a + h) + 12 \\ f(a + h) &= (a + h)(a + h) + 7(a + h) + 12 \\ f(a + h) &= a^2 + ah + ah + h^2 + 7a + 7h + 12 \\ f(a + h) &= a^2 + 2ah + h^2 + 7a + 7h + 12. \end{aligned}$$

If the expression has two of the functional letters (usually “ f ”), then the function must be written twice with the expression inside the parenthesis substituted for the independent variable. For example, $\frac{f(a + h) - f(a)}{h}$ means

subtract the function from itself substituting “ $a + h$ ” into the first function and “ a ” into the second function and put the whole expression over h :

$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{x^2 + 7x + 12}{h} - \frac{x^2 + 7x + 12}{h} \\ &= \frac{(a + h)^2 + 7(a + h) + 12 - [a^2 + 7a + 12]}{h} \\ &= \frac{(a + h)(a + h) + 7(a + h) + 12 - [a^2 + 7a + 12]}{h} && \text{foil \& distribute negative} \\ &= \frac{a^2 + ah + ah + h^2 + 7a + 7h + 12 - a^2 - 7a - 12}{h} \\ &= \frac{a^2 + 2ah + h^2 + 7a + 7h + 12 - a^2 - 7a - 12}{h} && \text{combine ah + ah} \\ &= \frac{a^2 - a^2 + 2ah + 7a - 7a + 12 - 12 + h^2}{h} && \text{group like terms} \\ &= \frac{2ah + h^2}{h} && \text{combine like terms} \\ &= \frac{h(2a + h)}{h} && \text{factor out an "h"} \\ &= 2a + h && \text{cancel the h} \end{aligned}$$