10.1 Conic Sections: Parabolas

The first portion of this chapter will discuss the various conics that are important in the world of mathematics. This section will examine the parabola. An example of a parabola that is used everyday by society is the headlamp on an automobile. The foil used as a reflector is in the shape of a 3-dimensional parabola. If you examine in the evening the light reflected by the headlamp, it is in the shape of a parabola. Mr. Burger mentioned that the general form for the equation of a parabola is:

$$x = cy^2 \text{ or } y = cx^2$$

where c is a rational number.

The vertex of the parabola $y = cx^2$ is at the origin, opening upward if $c > 0$ or opening downward if $c < 0$. The position of the focus is at $(0, 1/4c)$ with the equation of the directrix being $y = -1/4c$.

The vertex of the parabola $x = cy^2$ is at the origin, opening to the right if $c > 0$ or opening to the left if $c < 0$. The position of the focus is at $(1/4c, 0)$ with the equation of the directrix being $x = -1/4c$.

I, personally, would rather view the equation of a parabola with nice integer values, whenever possible. Therefore, by manipulating the equation so that c is on the other side of the equation and allowing that rational number to be represented by $4p$, we yield the following equations that represent a parabola:

$$y^2 = 4px \text{ or } x^2 = 4py$$

where p is a rational number.

The vertex of the parabola $x^2 = 4py$ is at the origin, opening upward if $p > 0$ or opening downward if $p < 0$. The position of the focus is at $(0, p)$ with the equation of the directrix being $y = -p$.

The vertex of the parabola $y^2 = 4px$ is at the origin, opening to the right if $p > 0$ or opening to the left if $p < 0$. The position of the focus is at $(p, 0)$ with the equation of the directrix being $x = -p$.

Example

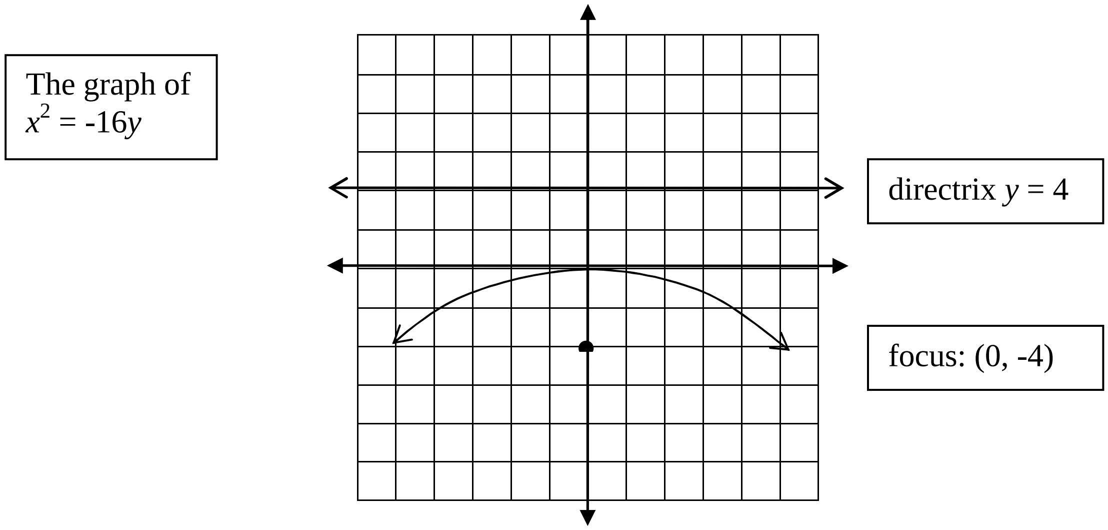
Graph $x^2 = -16y$, and locate the focus and directrix.

By examining the equation, we see that x is squared and therefore envision the graph opening either upward or downward. Since the value in front of y is negative we can deduce that the graph of this parabola is opening downward.

The vertex is located at (0, 0).

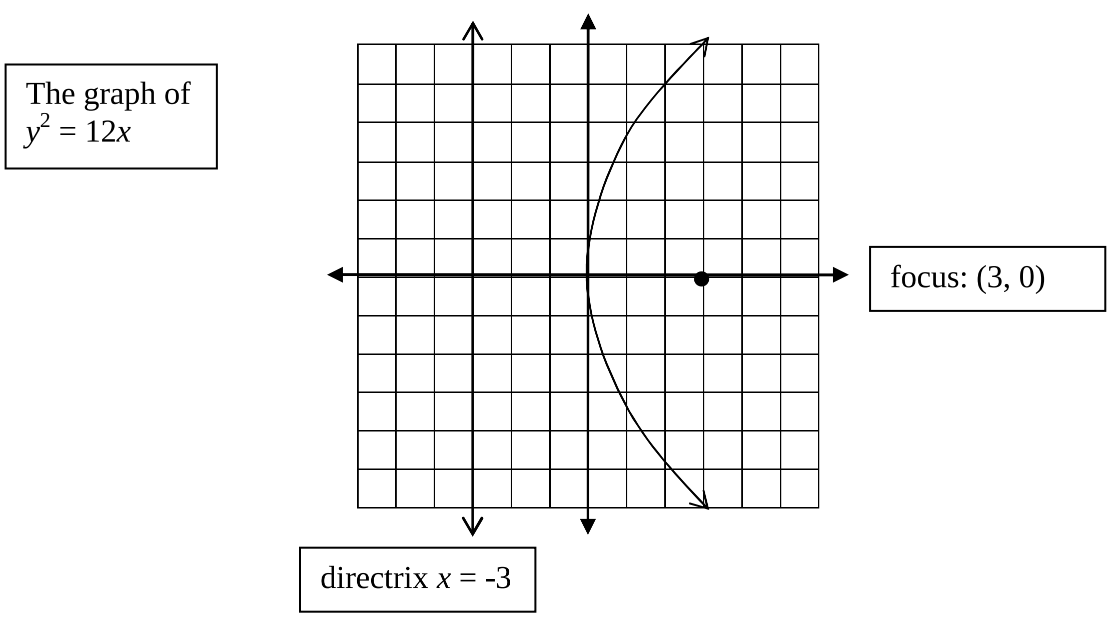
The length of the focus from the vertex is of length 4 units. By going in a downward direction 4 units from the vertex, we see that the focus is at (0, -4). By going in an upward direction 4 units from the vertex, we see that the equation of the directrix is $y = 4$.

The length across the focus is length 16. So, we can go to the right 8 units as well as to the left 8 units to determine two other points the parabola travels through.



Example
Graph $y^2 = 12x$, and locate the focus and directrix.

By examining the equation, we see that y is squared and therefore envision the graph opening either to the left or to the right. Since the value in front of x is positive we can deduce that the graph of this parabola is opening to the right.
The vertex is located at (0, 0).
The length of the focus from the vertex is of length 3 units. By going to the left 3 units from the vertex, we see that the focus is at (3, 0). By going to the left 3 units from the vertex, we see that the equation of the directrix is $x = -3$. The length across the focus is length 12. So, we can go to up 6 units as well as to the down 6 units to determine two other points the parabola travels through.



Try the following:

1. Graph $y^2 = -8x$, and locate the focus and directrix.

2. Graph $x^2 = 6y$, and locate the focus and directrix.

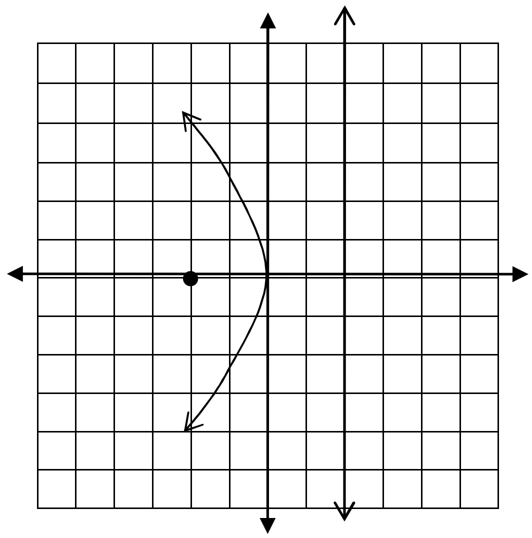
Answers:

1. Graph opens to the left.

The graph of $y^2 = -8x$

focus: (-2, 0)

directrix $x = 2$

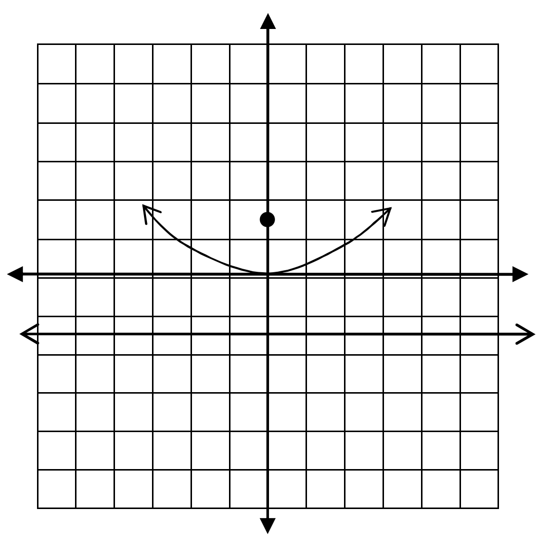


2. Graph opens upward.

The graph of $x^2 = 6y$

focus: (3, 0)

directrix $y = -6/4$
$y = -3/2$
$y = -1.5$



Let's look at determining the equation of a parabola given information about the parabola.

Example
Find the equation of a parabola having the origin as its vertex, the y axis as its axis of symmetry, and (-10, -5) on its graph. Find the coordinates of the focus and the equation of its directrix.

According to its axis of symmetry we know that the parabola will open either upward or downward. By plotting the given point (-10, -5) along with the vertex, we see that the graph of the parabola will open downward. The equation we are looking at describing is in the form $x^2 = 4py$. By substituting the coordinates of the point in for x and y we can deduce that the value of p is -5. Thus, the equation of the parabola is $x^2 = -20y$. The length of the focus is 5. Therefore, the focus is located at (0, -5). The equation of the directrix is $y = 5$.

Example
Find the equation of the parabola given directrix $x = 2$ and focus (-2, 0).

According to the direction of the directrix, we can deduce that the graph of this parabola will either open to the right or to the left. By plotting the focus, we can further deduce that the graph of this parabola will open to the right. We know that the position of the vertex to be located directly between the directrix and the focus. Therefore, the vertex is located at (0, 0). The length of the focus is 2.
Thus, the equation of the parabola is $y^2 = 8x$.

Try the following

1. Find the equation of a parabola having the origin as its vertex, the x axis as its axis of symmetry, and (4, -8) on its graph. Find the coordinates of its focus and the equation of its directrix.

2. Find the equation of a parabola having the origin as its vertex, the y axis as its axis of symmetry, and (2, -1) on its graph. Find the coordinates of its focus and the equation of its directrix.

3. Find the equation of the parabola given directrix $y = -4$ and focus (0, 4).

Answers

1. $y^2 = 16x$; focus: (4, 0) ; directrix: $x = -4$

2. $x^2 = -4y$; focus: (0, -1) ; directrix: $y = 1$

3. $x^2 = 16y$